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# Buyback Pricing of Durable Goods in Dual Distribution Channels

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In the U.S. automobile industry, manufacturers distribute products to consumers through dealers and rental agencies. By selling slightly used rental cars to consumers, rental agencies can compete directly against dealers in the sales market. To mediate this conflict between the two intermediaries, manufacturers launched buyback programs to repurchase used rental cars from rental agencies and redistribute them through dealers. Manufacturers have the option to precommit a buyback price to rental agencies at the time of initial sales or postpone the pricing decision to the time of repurchase. Using a two-period model with and without manufacturer competition in the rental market, we seek to understand how to manage buyback pricing to maximize manufacturers' profits in such dual distribution channels. Under precommitted buyback pricing, we show that the monopolist manufacturer's pricing decisions have a submodular property—when she increases the buyback price in the rental market, she would have to decrease the wholesale price in the sales market. This tradeoff is eliminated under postponed buyback pricing. We find that precommitted buyback pricing not only leads to zero buyback quantity in equilibrium due to a low buyback price but also yields a lower manufacturer profit than postponed buyback pricing. When manufacturers compete in the rental market, however, they may prefer precommitting their buyback prices in equilibrium if the competition is sufficiently intense. This is due to a strategic complementarity between manufacturers' buyback prices.

*Key words*: dual-channel supply chains, durable goods, buyback pricing, manufacturer competition

## 1. Introduction

Channel competition may arise when manufacturers distribute products through different channel intermediaries. Effectively managing the relationships with and mediating the conflicts between intermediaries are of key importance to manufacturers' profitability. In the U.S. automobile industry, manufacturers distribute products to consumers through dealers in the sales market and rental agencies in the rental market. Until the late 1980s, these two channels were *separate*: dealers were franchised to sell products in the sales market and rental agencies to rent in the rental market. Low sales in the consumer market prompted manufacturers to experiment with new channel structures. Initially, rental agencies were allowed to sell used rental cars to consumers. This so-called *overlapping* channel arrangement led to a large number of slightly used rental cars entering the sales market, thereby introducing a direct competition between rental agencies and dealers. This channel strategy backfired with strong opposition from dealers, and some of them even brought law suits against the manufacturers (Auto Rental News 1990*a*). In response to the channel conflict, manufacturers launched a *buyback* channel to sell the so-called *program* cars to rental agencies, repurchase them at a guaranteed price, and redistribute them through dealers. For example, major auto manufacturers in the United States, including Ford, GM, Chrysler, and Toyota USA, have similar repurchase programs for daily rental agencies.[1] When these rental agencies make initial purchase of program cars, auto manufacturers guarantee to buy them back for a set monthly or daily depreciation rate, provided that the cars are returned in the contracted condition and in the specified time period, which often ranges from 6 to 12 months. By doing this, the manufacturers essentially precommit a buyback price to rental agencies when they purchase new cars.

Buyback programs of auto manufacturers have been widely viewed by rental agencies as financially attractive because they can retire used rental fleets after a short period of time based on market conditions. This is especially beneficial for rental agencies when they purchase car models that depreciate quickly (see, e.g., Yopp and Eckhaus (2010)). By precommitting to a buyback price, manufacturers give rental agencies assurance of program cars' repurchase value, which presumably gives rental agencies incentives to purchase more new cars. However, does this necessarily improve manufacturers' profitability? As a natural alternative to precommitted buyback pricing, manufacturers can postpone the pricing decision to the time when they repurchase used rental cars from rental agencies. Postponement gives manufacturers the flexibility to adjust buyback pricing based on the supply of used cars in the market and thus on the actual residual value of used cars. It is not ex-ante clear how this flexibility in buyback pricing would affect manufacturers' profitability. Surprisingly, little has been done in the supply chain literature regarding how to manage buyback pricing of durable goods in dual distribution channels. Neither is it well understood how manufacturers' buyback prices would affect new-car orders from dealers and rental agencies.

To shed light on the preceding questions, we build a stylized model that captures the essential characteristics of the distribution channels of the U.S. automobile industry. Specifically, we consider a two-period model in which a monopolist manufacturer sells a durable product through two distribution channels: a dealer in the sales market and a rental agency in the rental market. The durable product depreciates in value over time, as reflected by a reduced consumer utility. In both periods, the manufacturer announces wholesale prices and the intermediaries submit orders. In the

[1] See the 2015 Ford Daily Rental Repurchase Program (www.fleet.ford.com), the GM 2015 Model Year Daily Rental National Purchase Program Guidelines (www.gmfleet.com), the 2014 Chrysler Group LLC Guaranteed Depreciation Program Rules (www.fleet.chrysler.com), and the 2016 Toyota National Daily Rental Repurchase Program (fleet.toyota.com).

second period, the rental agency may return used products to the manufacturer who pays a buyback price, and the manufacturer will then resell them to the dealer. We first study the monopolist manufacturer's optimal buyback pricing under two regimes – *precommitted* and *postponed* – the manufacturer announces the buyback price in the first period under the precommitted regime, but in the second period under the postponed regime.

We characterize the subgame perfect equilibrium of the two pricing regimes and compare their properties. Under the precommitted regime, we prove that the manufacturer's two-period total profit is submodular in the buyback price offered to the rental agency and the wholesale price offered to the dealer. In other words, if the manufacturer increases the buyback price in the rental market, she would have to decrease the wholesale price in the sales market. This tradeoff comes from the fact that buyback pricing exerts opposite effects on the two intermediaries: When the buyback price increases, it imposes a *positive direct effect* on the rental agency by increasing his order quantity, whereas it exerts a *negative indirect effect* on the dealer by reducing his order quantity. As a result, when the manufacturer offers an attractive (i.e., high) buyback price to the rental agency, she would have to offer an attractive (i.e., low) wholesale price to the dealer.

In equilibrium, it turns out that the manufacturer sets the buyback price so low that the rental agency does not return any used products to the manufacturer. Our analysis finds that precommitted buyback pricing makes it unprofitable for the manufacturer to induce a positive buyback quantity in equilibrium because the required buyback price would be higher than the equilibrium resale price charged to the dealer, thereby making the manufacturer lose money on each repurchased product. Under postponed buyback pricing, by contrast, the aforementioned tradeoff between the buyback price and the wholesale price is eliminated. This not only leads to a positive buyback quantity in equilibrium but also yields a higher profit for the manufacturer. Postponement gives the manufacturer a strategic advantage of setting the buyback price after the intermediaries have already made their first-period order decisions.

The higher profitability associated with postponed buyback pricing begs the question of why precommitted buyback pricing is widely implemented in practice by major U.S. automobile manufacturers. To seek answers to this question, we extend the base model to a setting where two manufacturers distribute imperfectly substitutable products through the same rental agency. The manufacturers simultaneously choose their pricing regime from two options, precommitted and postponed, at the beginning of the first period. By characterizing the equilibrium, we find both manufacturers may prefer precommitting their buyback prices when the product substitution factor is sufficiently large, i.e., when the competition in the rental market is sufficiently intense. Interestingly, in such an equilibrium, the buyback quantities are positive, contrasting the zero buyback quantity in the monopoly setting with precommitted buyback pricing. We attribute this equilibrium outcome to the strategic complementarity between manufacturers' buyback prices. Our numerical study indicates that the precommitment equilibrium arises for a large parameter space. This equilibrium outcome is consistent with the observation that major U.S. automobile manufacturers offer a predetermined depreciation schedule to rental agencies for program cars.

The remainder of the paper is organized into four sections. We review related literatures in §2, We lay out the monopoly model in §3 and analyze it in §4. We extend the model to incorporate differentiated competition in the rental market in §5 and conclude in §6.

## 2. Literature Review

To the best of our knowledge, how to manage buyback pricing of durable goods in dual distribution channels has not been explored in the literature. Our work lies at the intersection of two literatures: (1) management of dual-channel supply chains; and (2) pricing strategies in durable goods markets. Although both topics have drawn extensive interests from researchers in both operations and marketing, little has been done to link the two literatures. The widespread adoption of sales-rental dual-channel supply chains in durable goods industries, ranging from automobiles to industrial and construction equipments, underscores the need for more research in this area.

Papers in the literature of dual-channel supply chains can be roughly divided into two streams according to the channel structure. The first stream focuses on the structure consisting of a direct channel, owned by a manufacturer, and a retail channel, owned by an intermediary. The main research questions center on the implications of the direct channel introduced by a manufacturer and how to manage the resulting conflict between the direct channel and the retail channel. Both Chiang et al. (2003) and Tsay and Agrawal (2004) find that the direct channel may be beneficial to the retailer due to the associated wholesale price reduction by the manufacturer. Arya et al. (2007) observe a similar outcome even under the assumption that the manufacturer's direct channel does not affect the retailer's demand. Cattani et al. (2006) show that the manufacturer can exercise an equal-pricing strategy to optimize her own profit and improve the retailer's profit by committing to a direct-channel retail price that matches the retailer's price. Huang and Swaminathan (2009) show how pricing strategies might be different between pure e-tailers and dual channel retailers. Cai (2010) finds that both the manufacturer and the retailer may benefit from the direct channel when the retail channel has a sufficient cost advantage over the direct channel. Extending Arya et al. (2007)'s model to incorporate asymmetric information, Li et al. (2014, 2015) show that both the manufacturer and the retailer may "lose" due to the costly signaling behavior on the part of the retailer. Unlike the aforementioned papers, Ha et al. (2016) endogenize the product quality decision of the manufacturer in a dual-channel supply chain. They find manufacturer encroachment (by launching a direct channel) always makes the retailer worse off in a large variety of scenarios, which contrasts the retailer-favored outcomes found in previous studies.

The second stream of research on dual-channel supply chains considers two indirect distribution channels that are owned by two different intermediaries, for example, a dealer's sales channel and a rental agency's rental channel. Similar to the first stream, the main research questions revolve around the channel conflict resulting from channel competition. Purohit and Staelin (1994) and Purohit (1997) both consider three channel structures of the U.S. automobile industry, namely, separate, overlapping, and buyback. Purohit and Staelin (1994) find that total manufacturer sales of new cars are greatest for an overlapping channel, but dealer profits are greater in a buyback channel than in an overlapping channel. Unlike Purohit and Staelin (1994), who treat the rental agency as exogenous to the system, Purohit (1997) models each player separately, and finds that the overlapping channel structure maximizes the manufacturer's profit, whereas the buyback channel structure results in a lower manufacturer profit, but it serves to mediate the channel conflict by improving the dealer's profit.

Our model adopts the structure of the buyback channel introduced in Purohit (1997). There are two major features of our model that differentiates our paper. First, we compare two different buyback pricing regimes by allowing manufacturers to either precommit or postpone the announcement of the buyback price. The buyback pricing regime in Purohit (1997) corresponds to our postponed regime. Incorporating precommitted buyback pricing, we make our model closer to reality by reflecting the current practice in the U.S. automobile industry. Precommitted buyback pricing not only introduces *strategic tradeoffs between manufacturers' pricing decisions* on the wholesale of new cars and the buyback of used cars, but also induces *divergent strategic responses in the new-car orders of dealers and rental agencies* that are otherwise irrelevant in the postponed regime. Our analysis offers practical managerial insights regarding the management of buyback pricing of durable goods in dual-channel supply chains. Second, we incorporate manufacturer competition and derive the endogenous equilibrium choice of buyback pricing regime. Our observation of the precommitted equilibrium outcome provides a plausible explanation for the prevailing buyback pricing practice of major U.S. auto manufacturers.

Our work also contributes to the literature on durable goods markets. One important phenomenon observed in the classical durable goods literature, e.g., Coase (1972) and Bulow (1982), is the trend of decreasing prices over time, namely, the "time inconsistency" problem. Much of the subsequent literature on durable goods has primarily concerned with the decision of leasing versus selling, e.g., Bhaskaran and Gilbert (2005), Gilbert et al. (2014). We do not intend to review the entire durable goods literature here, but selectively review papers that study the role of intermediaries in the context of durable goods markets because this is the segment that our work is more closely related to. Purohit (1995) finds that a manufacturer prefers her intermediary to sell rather than to rent products to consumers, reversing the standard result that renting is preferred to selling. Desai et al. (2004) consider a durable goods market where renting is infeasible. They show that a manufacturer can get around the time inconsistency problem by precommitting to a two-part contract. Similarly, Arya and Mittendorf (2006) show that although channel decentralization introduces double marginalization, it convinces consumers that the price will stay high and thus alleviates the time inconsistency problem and encourages consumers to purchase early in the product's life cycle. Bhaskaran and Gilbert (2009) consider lease brokering, under which the manufacturer decides on a wholesale lease price, and the dealer adds his own margin and leases the product to consumers. They find that lease brokering eliminates the time inconsistency problem. Bhaskaran and Gilbert (2015) find that channel decentralization increases a manufacturer's willingness to invest in product durability when combined with selling, reversing the traditional result that leasing encourages a direct-selling manufacturer to invest in durability. Gumus et al. (2013) consider demand uncertainty and find that a higher consumer valuation of used products increases the likelihood that a monopolist manufacturer offers a returns contract to her retailer. Yin et al. (2010) find that the presence of multiple used goods markets induces frequent product upgrades by a monopolist manufacturer.

Unlike the aforementioned papers on durable goods, our work studies pricing strategies of durable goods in a dual-intermediary setting. The interactions between the intermediaries, i.e., a dealer and a rental agency, as well as their different responses to manufacturers' prices play an important role in our results. Moreover, our study demonstrates that competition between manufacturers can induce them to precommit buyback pricing to rental agencies in equilibrium, which is otherwise unprofitable and leads to zero buyback quantity in a monopolist setting. It is worth mentioning that most papers in the durable goods literature do not consider manufacturer competition. A notable exception is Desai and Purohit (1999), who consider a duopoly of durable goods manufacturers and address the leasing vs selling decision in a centralized channel setting (i.e., they do not model intermediaries). They show that in equilibrium the manufacturers either sell all their units or use a mix of leasing and selling.

## 3. The Model

**Dual Channel Setup.** We consider a manufacturer making durable goods that last for two periods and distributing them to consumers through her intermediaries: a dealer and a rental agency (see Figure 1). Although our analysis applies to any durable-good industry where rental and sales market coexist, in the remainder of the paper we will refer to this durable good as car. There are three types of cars: *new*, *program*, and *used* cars, which we denote by subscripts $n$, $p$, and $u$, respectively. New cars can be rented through the rental agency or sold through the dealer. Program cars refer to used rental cars that manufacturers buyback from the rental agency, while used cars refer to those pre-owned by consumers.

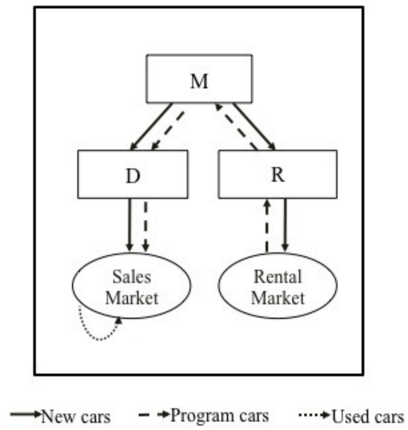


**Figure 1** **Buyback channel structure. The manufacturer, rental agency, and dealer are denoted by M, R, and D, respectively.**

We consider a two-period model. In the first period only new cars are available in the sales and rental markets, whereas in the second period new, program, and used cars coexist, as indicated in Figure 1. The manufacturer has the option to precommit to a buyback price when the rental agency purchases new cars or postpone the pricing decision to when she repurchases program cars from the rental agency. The sequence of events is as follows: In the first period the manufacturer chooses profit maximizing wholesale prices (and the buyback price, if precommitted buyback pricing is followed). The intermediaries decide how many new cars to buy from the manufacturer. In the second period, the manufacturer again chooses her wholesale price (and the buyback price, if postponed buyback pricing is followed). Then, the rental agency (dealer) decides how many program cars to return to (buy from) the manufacturer; and each decides how many new cars to buy from the manufacturer. Figure 2 illustrates the sequence of events.

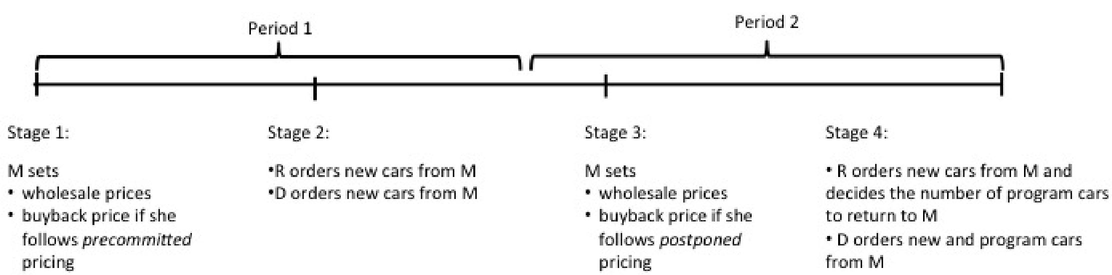


**Figure 2** **The sequence of events.**

**Consumer Demand.** Consumers are heterogenous in their valuation for a new car which is represented by $\phi = [0,1]$ where $\phi$ is distributed uniformly between 0 and 1. The total mass of consumers is normalized to 1. Following the literature (e.g., Purohit 1997, Desai and Purohit 1998), we assume that program cars pre-owned by the rental agency depreciate less than used cars pre-owned by consumers.[2] Accordingly, $0 \leq \theta_p \leq \theta_u \leq 1$ where $\theta_j$ is the depreciation rate of type $j$ car, $j \in \{p, u\}$. Without loss of generality, we assume that $\theta_p = \gamma\theta_u$ where $\gamma \in [0, 1]$ denotes the depreciation of program cars relative to used cars. Consumer valuation is $z = (1-\gamma\theta_u)\phi$ for program cars and $z = (1 - \theta_u)\phi$ for used cars. When a consumer with valuation $z$ purchases a car with price $p$, her utility is $z - p$. In any period each consumer purchases a car, if any, that provides her with the highest nonnegative utility. Given the new, program, and used car quantities, consumers make their purchasing decisions and this leads to the following inverse demand functions in the sales and rental markets:

$$p_{2u} = (1-\theta_u)(1-q_{2u}-q_{2p}-q_{2n}), \tag{1}$$

$$p_{2p} = p_{2u} + (\theta_u - \theta_p)(1-q_{2n}-q_{2p}), \tag{2}$$

$$p_{2n} = p_{2p} + \theta_p(1-q_{2n}), \tag{3}$$

$$p_{1n} = p_{2u} + (1-q_{1n}), \tag{4}$$

$$\bar{p}_{2p} = (1-\theta_p)(1-(\bar{q}_{2n} + (\bar{q}_{1n} - \bar{q}_v))) = (1-\theta_p)(1-(\bar{q}_{2n}+\bar{q}_{2p})), \tag{5}$$

$$\bar{p}_{2n} = \bar{p}_{2p} + \theta_p(1-\bar{q}_{2n}), \tag{6}$$

$$\bar{p}_{1n} = 1 - \bar{q}_{1n}, \tag{7}$$

where $p_{ij}$ ($\bar{p}_{ij}$) denotes the price and $q_{ij}$ ($\bar{q}_{ij}$) denotes the quantity of type $j$ cars in the sales (rental) market in period $i$ for $j \in \{u, p, n\}$ and $i \in \{1, 2\}$; and $\bar{q}_v$ denotes the quantity of program cars rental agency returns to the manufacturer in period 2. The derivation of these inverse demand functions follows from the durable goods literature (e.g., Purohit 1997); and therefore, omitted for brevity.

Next we formally state the manufacturer's and intermediaries' problems and solve for the subgame perfect equilibrium using backward induction. We model a quantity competition[3] between the intermediaries following the literature (e.g., Purohit 1997, Arya et al. 2007, Ha et al. 2016). We assume that there is no discounting in the manufacturer's and the intermediaries' decision problems. This is for analytical simplicity but adding discounting will not change our results qualitatively. For simplicity, we also normalize the manufacturer's production cost and the intermediaries' selling costs to zero. These assumptions are common in the durable goods literature (e.g., Purohit 1997). For clarity, we provide a list of notations in Table 1.

[2] To be eligible for buyback, program cars need to meet certain standards such as regular maintenance service records, original factory-installed equipments, completed warranty and recall repairs. These standards help explain the anecdotal evidence suggesting that program cars typically have a lower depreciation rate than used cars.

[3] While price competition is appropriate where capacity and production quantity can be adjusted easily (e.g., information goods), quantity competition is appropriate in modeling capital-intensive industries where production capacity is relatively fixed. For durable goods, manufacturing facility is expensive and can be adjusted only after considerable lead time. Thus modeling quantities as the decision variables of intermediaries is a reasonable approach.

| | |
|---|---|
| **Decision variables** | |
| $q_{ij}$ | quantities in the sales market for type $j=n$ car in period $i \in \{1,2\}$ and for type $j \in \{p,u\}$ car in period $i=2$ |
| $p_{ij}$ | prices in the sales market for type $j=n$ car in period $i \in \{1,2\}$ and for type $j \in \{p,u\}$ car in period $i=2$ |
| $\bar{q}_{ij}$ | quantities in the rental market for type $j=n$ car in period $i \in \{1,2\}$ and for type $j=p$ car in period $i=2$ |
| $\bar{q}_v$ | program cars the rental agency returns to the manufacturer in period 2 |
| $\bar{p}_{ij}$ | prices in the rental market for type $j=n$ car in period $i \in \{1,2\}$ and for type $j=p$ car in period $i=2$ |
| $w_i$ | wholesale price for new cars charged to the dealer in period $i \in \{1,2\}$ |
| $\bar{w}_i$ | wholesale price for new cars charged to the rental agency in period $i \in \{1,2\}$ |
| $\bar{v}$ | buyback price the manufacturer pays to the rental agency for program cars |
| $v$ | resale price the manufacturer charges the dealer for program cars (only relevant in period 2) |
| **Profits** | |
| $\Pi_i$ | the manufacturer's cumulative profit starting from period $i \in \{1,2\}$ |
| $\pi_i$ | the dealer's cumulative profit starting from period $i \in \{1,2\}$ |
| $\bar{\pi}_i$ | the rental agency's cumulative profit starting from period $i \in \{1,2\}$ |

**Table 1** **Summary of Notations for the Monopoly Model**

### 3.1 Precommitted Buyback Pricing Regime

Under the precommitted buyback pricing regime, the manufacturer chooses to precommit the buyback price at the time of initial sales of new rental cars. We denote this scenario by C.

**3.1.1 Second Period** The dealer decides how many new cars to buy ($q_{2n}$) at wholesale price $w_2$ and how many program cars to buy ($q_{2p}$) at price $v$ from the manufacturer, maximizing the total profit from new and program car sales:

$$\text{(P1)} \quad \max_{q_{2n}, q_{2p}} \pi_2 = q_{2n}(p_{2n} - w_2) + q_{2p}(p_{2p} - v). \tag{8}$$

Simultaneously, the rental agency decides how many program cars to return to the manufacturer ($\bar{q}_v$) at buyback price $\bar{v}$ and how many new cars to buy ($\bar{q}_{2n}$) at wholesale price $\bar{w}_2$:

$$\text{(P2)} \quad \max_{\bar{q}_{2n}, \bar{q}_v} \bar{\pi}_2 = \bar{q}_v \bar{v} + \bar{q}_{2p}\bar{p}_{2p} + \bar{q}_{2n}(\bar{p}_{2n} - \bar{w}_2). \tag{9}$$

where $\bar{q}_{2p} = \bar{q}_{1n} - \bar{q}_v$. The first component of the rental agency's profit comes from selling some of the program cars (that he purchased new in the first period) back to the manufacturer at buyback price $\bar{v}$. After selling these $\bar{q}_v$ units back, the rental agency rents the remaining program cars (i.e., $\bar{q}_{2p} = \bar{q}_{1n} - \bar{q}_v$) at price $\bar{p}_{2p}$, and hence the second component of his profit. Finally, the third component is the revenue from renting new cars ($\bar{q}_{2n}$) at price $\bar{p}_{2n}$ minus their cost.

In practice program cars are sold to the dealers through auctions and the manufacturers do not withhold any program cars (Auto Rental News 1990*b*). Consistent with this common practice in the automobile industry, we assume that all program cars repurchased from the rental agency are sold to the dealer. Therefore, resale price of program cars charged to the dealer ($v^*$) comes from the market equilibrium $\bar{q}_v^* = q_{2p}^*$. Given the optimal quantity decisions of intermediaries (i.e., $q_{2n}^*, q_{2p}^*, \bar{q}_{2n}^*, \bar{q}_v^*$), the manufacturer chooses new- and rental-car wholesale prices:

$$\text{(P3)} \quad \max_{\bar{w}_2, w_2} \Pi_2 = q_{2n}^* \, w_2 + \bar{q}_{2n}^* \, \bar{w}_2 + q_{2p}^* \, v^* - \bar{q}_v^* \, \bar{v} \tag{10}$$

$$= q_{2n}^* \, w_2 + \bar{q}_{2n}^* \, \bar{w}_2 + \bar{q}_v^* \, (v^* - \bar{v}). \tag{11}$$

The manufacturer's profit comes from the sale of new and rental cars, in addition to her profit from program car trading, i.e., purchasing program cars from the rental agency at buyback price $\bar{v}$ and selling them to the dealer at resale price $v^*$.

**3.1.2 First Period** The dealer decides how many new cars to buy ($q_{1n}$) at price $w_1$, whereas the rental agency decides how many new cars to buy ($\bar{q}_{1n}$) at price $\bar{w}_1$ from the manufacturer. Therefore, the dealer's first period problem is:

$$\text{(P4)} \quad \max_{q_{1n}} \pi_1 = \pi_2^* + q_{1n}(p_{1n} - w_1), \tag{12}$$

while the rental agency's first period problem is:

$$\text{(P5)} \quad \max_{\bar{q}_{1n}} \bar{\pi}_1 = \bar{\pi}_2^* + \bar{q}_{1n}(\bar{p}_{1n} - \bar{w}_1). \tag{13}$$

Finally, the manufacturer chooses new- and rental-car wholesale prices ($\bar{w}_1, w_1$) as well as the buyback price $\bar{v}$ to maximize her total profit in the two periods:

$$\text{(P6)} \quad \max_{\bar{w}_1, w_1, \bar{v}} \Pi_1 = \Pi_2^* + q_{1n}^* \, w_1 + \bar{q}_{1n}^* \, \bar{w}_1 \tag{14}$$

### 3.2 Postponed Buyback Pricing Regime

Under postponed buyback pricing regime, the manufacturer postpones the buyback price announcement to the second period, (as opposed to precommitting to buyback price at the time of initial sales of new cars in the first period). We denote this scenario by N. The intermediaries' problems in each period remain the same as in §3.1. In the second period, the manufacturer maximizes her profit $\Pi_2 = q_{2n}^* \, w_2 + \bar{q}_{2n}^* \, \bar{w}_2 + \bar{q}_v^* ( v^* - \bar{v})$ by choosing wholesale prices ($w_2, \bar{w}_2$) as well as the buyback price for program cars ($\bar{v}$). In the first period, the manufacturer maximizes her profit $\Pi_1 = \Pi_2^* + q_{1n}^* \, w_1 + \bar{q}_{1n}^* \, \bar{w}_1$ by setting the wholesale prices ($\bar{w}_1, w_1$).

## 4. Equilibrium Analysis: The Monopoly Model

In this section, we characterize the subgame perfect equilibrium under precommitted and postponed buyback pricing regimes in §4.1 and §4.2, respectively. Then, in §4.3 we characterize the manufacturer's preference on the pricing regime. All proofs are relegated to the appendix.

Recall that the second period intermediary problems do not depend on the manufacturer's buyback pricing regime. Therefore, before proceeding with the analysis under each pricing regime, we solve the intermediaries' second period problems. Solving the optimality conditions for the dealer's problem in (P1) and the rental agency's problem in (P2) simultaneously (details are provided in Appendix A), we characterize the Nash equilibrium:

$$q_{2n}^* = \frac{\gamma\theta_u - w_2 + v}{2\gamma\theta_u} \tag{15}$$

$$\bar{q}_{2n}^* = \frac{1-\bar{w}_2}{2} - (1-\gamma\theta_u)(\bar{q}_{1n} - \bar{q}_v^*) \tag{16}$$

$$q_{2p}^* = \frac{(1-\gamma\theta_u)w_2 - \gamma\theta_u(1-\theta_u)q_{1n} - v}{2\gamma\theta_u(1-\gamma\theta_u)} \tag{17}$$

$$\bar{q}_v^* = \max\left\{\frac{2\gamma\theta_u\bar{q}_{1n} - 2\gamma^2\theta_u^2\bar{q}_{1n} + \gamma\theta_u\bar{w}_2 - \bar{w}_2 + \bar{v}}{2\gamma\theta_u(1-\gamma\theta_u)}, 0\right\} \tag{18}$$

Recall that the manufacturer does not withhold any program cars (i.e., $\bar{q}_v^* = q_{2p}^*$ and $v^*$ comes from this equilibrium). This implies that in the equilibrium there are two possible outcomes: First, the buyback quantity is *positive*, if both the dealer and the rental agency want to trade program cars through the manufacturer's buyback program (i.e., $\bar{q}_v^* = q_{2p}^* > 0$). We call this positive-buyback outcome. Second, the buyback quantity is *zero*, if there is no trade of program cars (i.e., $\bar{q}_v^* = q_{2p}^* = 0$). We call this zero-buyback outcome. Next, for each of the two possible outcomes, we characterize each player's optimal decisions under each pricing regime.

## 4.1 Precommitted Buyback Pricing

**4.1.1 Second Period** If the manufacturer precommits the buyback price in the first period, in the second period she sets wholesale prices $(w_2, \bar{w}_2)$ to maximize her profit, i.e., solving the optimization problem (P3).

For the zero-buyback outcome (i.e., $\bar{q}_v^* = 0$), the optimal prices are

$$\bar{w}_2^* = \frac{1 - 2(1-\gamma\theta_u)\bar{q}_{1n}}{2}, \tag{19}$$

$$w_2^* = \frac{1 - (1-\theta_u)q_{1n}}{2}, \tag{20}$$

whereas, for the positive-buyback outcome (i.e., $\bar{q}_v^* > 0$), we find

$$\bar{w}_2^* = \frac{\gamma\theta_u(1-\theta_u)q_{1n} + 4\gamma\theta_u(1-\gamma\theta_u)\bar{q}_{1n} + \gamma\theta_u + 4\bar{v}}{2(2-\gamma\theta_u)}, \tag{21}$$

and $w_2^*$ remains the same as in (20).

Comparing (19) with (21), observe the difference in rental-car wholesale prices $\bar{w}_2^*$ under zero- and positive-buyback outcomes. Under the former (i.e., when $\bar{q}_v^* = 0$), $\bar{w}_2^*$ decreases as the rental agency's first-period order quantity $\bar{q}_{1n}$ increases. This is because the manufacturer has to offer a lower wholesale price to make the purchase of new cars attractive to the rental agency if he already has a large number of used rental cars carried over from the first period. Under the latter (i.e., when $\bar{q}_v^* > 0$), however, $\bar{w}_2^*$ increases as $\bar{q}_{1n}$ and/or $q_{1n}$ increases. This is because, when there are many program cars in the rental market eligible for buyback (i.e., $\bar{q}_{1n}$ is high) and/or many used cars in the sales market (i.e., $q_{1n}$ is high), trading program cars in the second period is not attractive for the manufacturer. In response, the manufacturer increases $\bar{w}_2^*$ to make keeping the program cars more attractive for the rental agency than replacing them by new cars.

**4.1.2 First Period** We solve the dealer's problem (P4) and the rental agency's problem (P5) simultaneously.

For the zero-buyback outcome (i.e., $\bar{q}_v^* = 0$), the optimal quantities are $q_{1n}^* = \frac{13-5\theta_u-8w_1}{(\theta_u+3)(9-5\theta_u)}$ and $\bar{q}_{1n}^* = \frac{7-3\gamma\theta_u-4\bar{w}_1}{2(7-\gamma^2\theta_u^2-2\gamma\theta_u)}$. Confirming intuition, $q_{1n}^*$ ($\bar{q}_{1n}^*$) is decreasing in $w_1$ ($\bar{w}_1$) and does not depend on the other decision variables.

For the positive-buyback outcome (i.e., $\bar{q}_v^* > 0$), the expressions for $q_{1n}^*$ and $\bar{q}_{1n}^*$ are tedious and thus presented in the appendix for brevity. With the following proposition, we show how these quantities change in the manufacturer's first-period prices.

PROPOSITION 1. *Under precommitted buyback pricing, when the equilibrium buyback quantity is positive (i.e., $\bar{q}_v^* > 0$), in the first period*

*i. the dealer's order quantity decreases in both the buyback price and the new-car wholesale price but increases in the rental-car wholesale price (i.e., $\frac{\partial q_{1n}^*}{\partial \bar{v}} < 0$, $\frac{\partial q_{1n}^*}{\partial \bar{w}_1} > 0$, and $\frac{\partial q_{1n}^*}{\partial w_1} < 0$), and*

*ii. the rental agency's order quantity increases in the buyback price, but decreases in both the new- and rental-car wholesale prices (i.e., $\frac{\partial \bar{q}_{1n}^*}{\partial \bar{v}} > 0$, $\frac{\partial \bar{q}_{1n}^*}{\partial \bar{w}_1} < 0$, $\frac{\partial \bar{q}_{1n}^*}{\partial w_1} < 0$).*

Proposition 1 confirms the intuition that the dealer purchases fewer cars (i.e., $q_{1n}^*$ decreases) as the new-car wholesale price $w_1$ increases. Furthermore, $q_{1n}^*$ also decreases as the buyback price $\bar{v}$ increases or the rental-car wholesale price $\bar{w}_1$ decreases. The intuition goes as follows: Conditions leading to high rental car sales in the first period (such as high $\bar{v}$ or low $\bar{w}_1$) result in ample amount of program cars in the second period, and thus the dealer can purchase these program cars from the manufacturer at a low price. As a result, the dealer decreases his first period order quantity to limit the used cars in the second period and to mitigate the competition with his future program car sales.

Finally, Proposition 1 also shows that the rental agency purchases fewer cars (i.e., $\bar{q}_{1n}^*$ decreases) as the new-car wholesale price $w_1$ increases. That is because when $w_1$ is high, the dealer purchases fewer new cars (i.e., $q_{1n}^*$ decreases), leading to a lower second period rental-car wholesale price $\bar{w}_2$ (see (21) and the following discussion). From the rental agency's view point, this makes purchasing cars in the second period more attractive than purchasing them in the first period and thus $\bar{q}_{1n}^*$ decreases.

Proposition 1 has characterized how the intermediaries respond to the manufacturer's pricing decisions. The next question is how should the manufacturer optimize her buyback price $\bar{v}$ while optimizing her wholesale prices $(\bar{w}_1, w_1)$? To this end, we solve the manufacturer's first-period problem (P6). The following proposition characterizes the relationship between the manufacturer's pricing decisions.

PROPOSITION 2. *Under precommitted buyback pricing, when the equilibrium buyback quantity is positive (i.e., $\bar{q}_v^* > 0$), the manufacturer's first-period cumulative profit $\Pi_1$ is supermodular in $(\bar{v}, \bar{w}_1)$ and submodular in $(\bar{v}, w_1)$.*

Proposition 2 shows that when the manufacturer announces a higher buyback price $(\bar{v})$, she would also choose a lower new-car wholesale price $(w_1)$. That is because a higher buyback price would incentivize the rental agency to increase his first-period order quantity while causing the dealer to decrease his (cf. Proposition 1). In order to alleviate this negative effect of higher buyback price on the dealer's first-period order, the manufacturer offers the dealer an attractive wholesale price (i.e., lower $w_1$). This interplay of prices makes the manufacturer face a *tradeoff* between inducing more orders from the rental agency and inducing more orders from the dealer. As a result, achieving a positive buyback outcome would be expensive for the manufacturer. The next two propositions illustrate that the manufacturer finds it optimal to set a low buyback price such that a zero buyback outcome arises in equilibrium.

PROPOSITION 3. *Under precommitted buyback pricing, $\forall \bar{v}$ such that $\bar{v} \le \bar{v}_T = \frac{(3\gamma\theta_u + 7)(\gamma\theta_u - 1)^2}{8(3-\gamma^2\theta_u^2)}$, a zero-buyback equilibrium (i.e., $\bar{q}_v^* = 0$) arises. Furthermore, $\exists\, \bar{v}^* > \bar{v}_T$ that yields a positive-buyback equilibrium (i.e., $\bar{q}_v^* > 0$). Also, $\frac{\partial \bar{v}_T}{\partial \theta_u} \le 0$ and $\frac{\partial \bar{v}_T}{\partial \gamma} \le 0$.*

Proposition 3 suggests that if the manufacturer announces a buyback price lower than the threshold $\bar{v}_T$, the rental agency does not find it attractive to return cars at such a low price. The threshold $\bar{v}_T$ decreases as the depreciation of used cars $\theta_u$ or the depreciation of program cars relative to used cars $\gamma$ increases. That is because the rental agency would prefer returning more depreciated cars at a lower buyback price. The proposition also shows that announcing a buyback price $\bar{v}^*$ that is greater than the threshold $\bar{v}_T$ yields a positive-buyback outcome in the equilibrium $(\bar{q}_v^* > 0)$. Therefore, in the first period the manufacturer has two options, i.e., choosing a low buyback price leading to zero buyback outcome or choosing a high buyback price leading to a positive-buyback outcome. Next we identify the manufacturer's optimal choice.

PROPOSITION 4. *Under precommitted buyback pricing, the manufacturer is always better off offering $\bar{v} \le \bar{v}_T$ so that $\bar{q}_v^* = 0$.*

Proposition 4, interestingly, shows that the manufacturer would always prefer choosing a low enough buyback price to effectively make the buyback program mute. Following Proposition 2 we already discussed the tradeoff between the buyback price and the wholesale price that makes achieving a positive-buyback outcome expensive. Next corollary gives us another reason.

COROLLARY 1. *Under precommitted buyback pricing, when the equilibrium buyback quantity is positive (i.e., $\bar{q}_v^* > 0$), the buyback price the manufacturer pays to the rental agency is always higher than the resale price charged to the dealer (i.e., $\bar{v}^* > v^*$).*

Corollary 1 shows that the manufacturer would have to lose money for each program car she buys back from the rental agency and sells through the dealer, and the buyback program would have been a loss center for the manufacturer.

### 4.2 Postponed Buyback Pricing

Under the postponed buyback pricing regime, the buyback price is not precommitted at the time of initial purchase but rather announced in the second period. Recall that the solution to the intermediaries' second period problems is given in (15)-(18). We find that, similar to our results under precommitted buyback pricing, if the manufacturer chooses a small buyback price (i.e., $\bar{v}^* < \bar{v}_T$), then the zero-buyback outcome arises (i.e., $q_{2p}^* = \bar{q}_v^* = 0$). The solution here remains the same as in §4.1, because when the buyback quantity is zero, pricing regime becomes irrelevant. Thus we do not repeat the details here.

We continue to analyze the players' optimal decisions through backward induction. In the second period, the manufacturer maximizes her profit $\Pi_2 = q_{2n}^* w_2 + \bar{q}_{2n}^* \bar{w}_2 + q_{2p}^* v^* - \bar{q}_v^* \bar{v}$ by choosing wholesale prices $(w_2, \bar{w}_2)$ as well as the buyback price $\bar{v}$. From the optimality conditions, we find that the wholesale prices are the same as in (19) and (20), whereas the buyback price is

$$\bar{v}^* = \frac{2 - 2\theta_u \gamma - \theta_u \gamma (1-\theta_u) q_{1n} - 2(2+\gamma\theta_u)(1-\gamma\theta_u)\bar{q}_{1n}}{4}. \tag{22}$$

This expression suggests that the buyback price $\bar{v}^*$ increases as first-period new- and/or rental-car sales decrease. This is because when there is fewer used cars (as a result of lower $q_{1n}$) or program cars (as a result of lower $\bar{q}_{1n}$), the buyback program becomes more attractive to the manufacturer; and therefore she offers a higher buyback price in the second period.

In the first period, the intermediaries maximize their profits by making the optimal order decisions. The expressions for $q_{1n}^*$ and $\bar{q}_{1n}^*$ are tedious and relegated to the appendix for brevity. Next result identifies how these quantities depend on the wholesale prices.

PROPOSITION 5. *Under postponed buyback pricing, when the equilibrium buyback quantity is positive (i.e., $\bar{q}_v^* > 0$), in the first period* *i. the dealer's order quantity increases in the rental-car wholesale price and decreases in the new-car wholesale price (i.e.,* $\frac{\partial q_{1n}^*}{\partial \bar{w}_1} > 0$ *and* $\frac{\partial q_{1n}^*}{\partial w_1} < 0$*) , and*

*ii. the rental agency's order quantity decreases in the rental-car wholesale price and increases in the new-car wholesale price (i.e.,* $\frac{\partial \bar{q}_{1n}^*}{\partial \bar{w}_1} < 0$ *and* $\frac{\partial \bar{q}_{1n}^*}{\partial w_1} > 0$*).*

Comparing Proposition 5 with Proposition 1, we observe that the changes in the dealer's first period order quantity $q_{1n}^*$ with respect to the wholesale prices are the same as under the precommitted pricing regime. However, the effect of new-car wholesale price ($w_1$) on rental-car order quantity ($\bar{q}_{1n}$) is completely reversed. Specifically, under postponed buyback pricing, the rental agency would purchase *more* new cars (i.e., $\bar{q}_{1n}^*$ increases) as $w_1$ increases. This is because as the new-car wholesale price $w_1$ increases, the dealer's order quantity of new cars would decrease, and the buyback price announced in the second period would increase (see (22)). As a result, the rental agency would increase his first-period order quantity because he can return these cars at a high buyback price in the second period.

Finally, the manufacturer maximizes her cumulative first period profit $\Pi_1 = \bar{q}_{1n}^*(\bar{w}_1, w_1)\,\bar{w}_1 + q_{1n}^*(\bar{w}_1, w_1)\,w_1 + \Pi_2^*$ by setting the wholesale prices $(\bar{w}_1, w_1)$. We relegate the details to the appendix for brevity. Next, we compare zero- and positive-buyback outcomes and identify the conditions under which each outcome arises in equilibrium.

Proposition 6. *Under postponed buyback pricing,* $\forall \bar{v}$ *such that* $\bar{v} \leq \bar{v}_T$*, where* $\bar{v}_T$ *is given in Proposition 4, an equilibrium with* $\bar{q}_v^* = 0$ *arises. Furthermore,* $\exists\, \bar{v}^* > \bar{v}_T$ *that yields an equilibrium with* $\bar{q}_v^* > 0$*.*

Proposition 6 shows that, the minimum buyback price that gives a positive-buyback outcome is the same as that under the precommitment regime. Next proposition identifies the manufacturer's optimal choice.

Proposition 7. *Under postponed buyback pricing, the manufacturer is always better off offering* $\bar{v}^*$ *so that* $\bar{q}_v^* > 0$*.*

In the proof of Proposition 7, we show that the optimal buyback price paid to the rental agency is smaller than the resale price charged to the dealer (i.e., $\bar{v}^* < v^*$). Therefore, the manufacturer makes profit from the transaction of program cars, and thus the optimal buyback quantity is positive. Note that this is just the opposite of what we find under precommitted buyback pricing (see Corollary 1). The reason why the manufacturer is able to provide a lower buyback price under the postponement regime can be explained as follows: When the buyback price is determined in the second period, it does not affect the intermediaries' first-period order quantities, thereby eliminating the tradeoff in the manufacturer's pricing decisions under the precommitment regime (i.e., Proposition 2). This gives the manufacturer a chance to announce a lower buyback price than she would have under the precommitment regime.

### 4.3 The Manufacturer's Preferred Buyback Pricing Regime

Propositions 4 and 7 together show that a completely different equilibrium arises depending on which buyback pricing regime is followed by the manufacturer. Specifically, if the manufacturer postpones the announcement of buyback price until the second period, the optimal buyback quantity is positive. However, when the manufacturer precommits buyback price at the time of initial sales of new rental cars in the first period, then the optimal buyback quantity becomes zero. Next we characterize the manufacturer's preference over the two pricing regimes.

PROPOSITION 8. *The manufacturer's total two-period profit is higher under the postponed buyback pricing regime than under the precommitted buyback pricing regime, i.e.,* $\Pi_1^{N*} \geq \Pi_1^{C*}$.

This proposition shows that, the manufacturer always prefers postponed buyback pricing as precommitment hurts her profit. In other words, the attractiveness of a buyback program critically depends on whether the buyback price is precommitted at the time of initial purchase of new rental cars or not. By precommitting to the buyback price, the manufacturer gives the intermediaries a strategic advantage to adjust their first-period order quantities, which ends up hurting the manufacturer's profit. This result seems to contradict the prevalent use of precommitted buyback pricing in the U.S. automobile industry (e.g., Ford's repurchase programs). To further investigate this problem, next we extend the base model to introduce competition in the rental market.

## 5. Manufacturer Competition in the Rental Market

In this section, we extend our earlier model by considering competition between two manufacturers in the rental market. The manufacturers, denoted by superscripts $a$ and $b$, sell to the same rental agency but each manufacturer has her own dealer $k$ for $k \in \{a, b\}$. In other words, manufacturers have separate sales markets but compete in the same rental market (See Figure 3). This captures the setting in the automobile industry where consumers have much less brand loyalty when renting a car than when buying a car.[4] It is also consistent with the channel structure that rental agencies often carry several manufacturers' products. For example, Hertz carries several brands, such as Ford, Toyota, and Honda. By contrast, dealers usually only sell a single manufacturer's products.

Each manufacturer can follow the *precommitted* or *postponed* buyback pricing regime. We denote the buyback pricing regime manufacturer $k$ adopts by $s_k \in \{C, N\}$, where $C$ and $N$ indicate precommitted and postponed buyback pricing regimes, respectively (To help remember, C indicates "committed" while N indicates "not committed."). Therefore, there are four possible equilibrium outcomes of the manufacturers' choices of buyback pricing regime, denoted by $(s_a, s_b)$ where $(s_a, s_b) \in \{(C, N), (N, C), (N, N), (C, C)\}$. Next we present the problem formulation under competition for each possible equilibrium outcome of buyback pricing regime.

[4] Many car brands have high customer loyalty in the sales market, indicating a less intensive competition in the sales market than in the rental market. For example, 63% of all Ford Fiesta owners, when they decide to buy a new car, return to Ford to buy a new one (Forbes 2011).

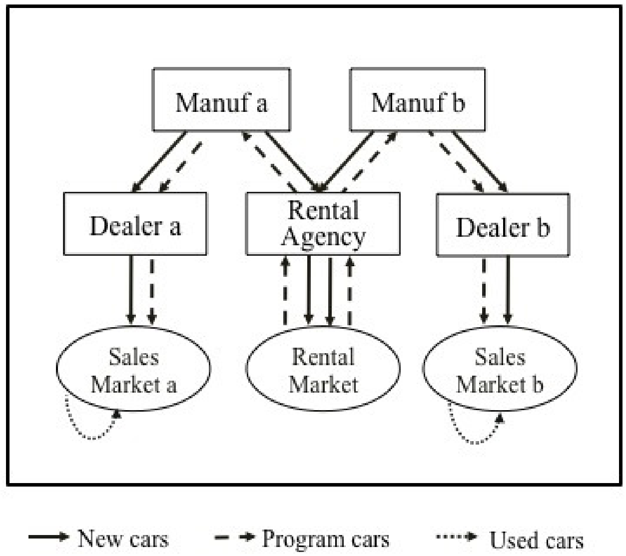


**Figure 3** **Buyback channel structure when there are two competing manufacturers in the rental market**

### 5.1 The Model

To capture the differentiated competition in the rental market, we follow the common approach in the operations and marketing literatures by introducing a parameter representing the degree of substitutability between the two manufacturers' products, $\beta \in [0,1)$, and modify the inverse demand functions in (5)-(7) accordingly (e.g., Trivedi 1998 and Feng and Lu 2013). We retain the same structure for the inverse demand functions in the sales market as the monopoly setting (e.g., those given in (1)-(4)). Here, we present the inverse demand functions in the rental market for manufacturer $a$ (the inverse demand functions for manufacturer $b$ have similar structures and are omitted here for brevity):

$$\bar{p}^a_{2p} = (1-\theta_p)(1-(\bar{q}^a_{2n}+\bar{q}^a_{2p})-\beta(\bar{q}^b_{2n}+\bar{q}^b_{2p})), \tag{23}$$

$$\bar{p}^a_{2n} = \bar{p}^a_{2p}+\theta_p(1-\bar{q}^a_{2n}-\beta\bar{q}^b_{2n}), \tag{24}$$

$$\bar{p}^a_{1n} = 1-\bar{q}^a_{1n}-\beta\bar{q}^b_{1n}, \tag{25}$$

where $\bar{p}^k_{ij}$ denotes the price and $\bar{q}^k_{ij}$ denotes the quantity for manufacturer $k$'s type-$j$ cars in the rental market in period $i$, for $k \in \{a,b\}$, $j \in \{u,p,n\}$, and $i \in \{1,2\}$. When $\beta = 0$, there is no substitutability and thus no competition exists in the rental market. Therefore, our earlier analysis for the monopoly setting is equivalent to the case where manufacturer cars are not substitutable in the rental market (i.e., $\beta = 0$).

Note that the structure of the dealer and manufacturer problems remain the same as in the monopoly setting. However, the problem formulation differs for the rental agency as he now carries cars from two manufacturers. Therefore, in formally stating each player's problem next, when necessary we refer to §3 for the dealer and manufacturer problems for brevity, but provide the details for the rental agency's problem. We also provide a list of notations in Table 2 for convenience.

| | |
|---|---|
| **Parameters** | |
| $\beta$ | Substitutability of the two manufacturers' cars in the rental market |
| $\gamma$ | depreciation of program cars relative to used cars |
| $\theta_u$ | depreciation of used cars |
| **Decision variables** | |
| $q_{ij}^k$ | quantities in the sales market for type $j=n$ car in period $i \in \{1,2\}$ and for type $j \in \{p,u\}$ car of manufacturer $k \in \{a,b\}$ in period $i=2$ |
| $p_{ij}^k$ | prices in the sales market for type $j=n$ car in period $i \in \{1,2\}$ and for type $j \in \{p,u\}$ car of manufacturer $k \in \{a,b\}$ in period $i=2$ |
| $\bar{q}_{ij}^k$ | quantities in the rental market for type $j=n$ car in period $i \in \{1,2\}$ and for type $j=p$ car of manufacturer $k \in \{a,b\}$ in period $i=2$ |
| $\bar{p}_{ij}^k$ | prices in the rental market for type $j=n$ car in period $i \in \{1,2\}$ and for type $j=p$ car of manufacturer $k \in \{a,b\}$ in period $i=2$ |
| $\bar{q}_v^k$ | program cars rental agency returns to manufacturer $k \in \{a,b\}$ in period 2 |
| $w_i^k$ | wholesale price for new cars charged to dealer $k \in \{a,b\}$ in period $i \in \{1,2\}$ |
| $\bar{w}_i^k$ | wholesale price for new rental cars charged to the rental agency in period $i \in \{1,2\}$ by manufacturer $k \in \{a,b\}$ |
| $\bar{v}^k$ | buyback price paid to the rental agency for program cars by manufacturer $k \in \{a,b\}$ |
| $v^k$ | resale price charged to dealer $k \in \{a,b\}$ for program cars (only relevant in period 2) |
| **Profits** | |
| $\Pi_i^k$ | manufacturer $k$'s cumulative profit starting from period $i \in \{1,2\}$ for $k \in \{a,b\}$ |
| $\pi_i^k$ | dealer $k$'s cumulative profit starting from period $i \in \{1,2\}$ for $k \in \{a,b\}$ |
| $\bar{\pi}_i$ | rental agency's cumulative profit starting from period $i \in \{1,2\}$ |

**Table 2 Summary of Notations for the Competition Model**

#### 5.1.1 Both Manufacturers Choose Precommitted Buyback Pricing: (C,C) Equilibrium

In a (C,C) equilibirum, in the second period, each dealer $k$ solves his problem as given in (P1). Simultaneously, the rental agency decides how many program cars to return to manufacturer $k$ ($\bar{q}_v^k$) at buyback price $\bar{v}^k$ and how many new rental cars to buy from manufacturer $k$ ($\bar{q}_{2n}^k$) at wholesale price $\bar{w}_2^k$:

$$\text{(P7)} \quad \max_{\bar{q}_{2n}^a, \bar{q}_{2n}^b, \bar{q}_v^a, \bar{q}_v^b} \bar{\pi}_2 = \sum_{k \in \{a,b\}} \left( \bar{q}_v^k \bar{v}^k + (\bar{q}_{1n}^k - \bar{q}_v^k) \bar{p}_{2p}^k + \bar{q}_{2n}^k (\bar{p}_{2n}^k - \bar{w}_2^k) \right). \tag{26}$$

As before, all program cars repurchased from the rental agency are sold to the respective dealers; namely, the manufacturers do not withhold any program cars. Therefore, the resale price charged to dealer $k$ ($v^{k*}$) comes from equilibrium $\bar{q}_v^{k*} = q_{2p}^{k*}$ for $k \in \{a,b\}$. Given the optimal quantity decisions of the intermediaries (i.e., $q_{2n}^{k*}, q_{2p}^{k*}, \bar{q}_{2n}^{k*}, \bar{q}_v^{k*}$), each manufacturer simultaneously chooses new and rental-car wholesale prices by solving her problem as given in (P3).

In the first period, each dealer $k$ solves his problem as given in (P4), whereas the rental agency decides how many new rental cars to buy ($\bar{q}_{1n}^k$) at price $\bar{w}_1^k$ from manufacturer $k$ for $k \in \{a,b\}$ as follows:

$$\text{(P8)} \quad \max_{\bar{q}_{1n}^a, \bar{q}_{1n}^b} \bar{\pi}_1 = \bar{\pi}_2^* + \sum_{k\in\{a,b\}} \bar{q}_{1n}^k (\bar{p}_{1n}^k - \bar{w}_1^k). \tag{27}$$

Finally, manufacturer $k$ maximizes her profit as given in (P6).

#### 5.1.2 Both Manufacturers Choose Postponed Buyback Pricing: (N,N) Equilibrium

When both manufacturers choose to postpone their buyback price announcements (i.e., in an (N, N) equilibrium), the intermediaries' problems in each period remain the same as in §5.1.1. In the second period, manufacturer $k$ maximizes her profit $\Pi_2^k = q_{2n}^{k*} w_2^k + \bar{q}_{2n}^{k*} \bar{w}_2^k + \bar{q}_v^{k*} (v^{k*} - \bar{v}^k)$ by choosing wholesale prices $(w_2^k, \bar{w}_2^k)$ as well as the buyback price for program cars $(\bar{v}^k)$. In the first period, manufacturer $k$ maximizes her profit $\Pi_1^k = \Pi_2^{k*} + q_{1n}^{k*} w_1^k + \bar{q}_{1n}^{k*} \bar{w}_1^k$ choosing only the wholesale prices $(\bar{w}_1^k, w_1^k)$.

#### 5.1.3 Asymmetric Choices of Buyback Pricing Regime: (N,C) or (C,N) Equilibria

If only one manufacturer precommits the buyback price at the time of initial sales of new rental cars whereas the other one waits until the time of repurchase to announce her buyback price, we call this (C,N) (or, (N,C)) equilibrium. The intermediaries' problems remain the same as in §5.1.1. The problem of the manufacturer who precommits buyback price is the same as in §5.1.1 whereas the problem of the manufacturer who postpones buyback pricing is the same as in §5.1.2.

### 5.2 Equilibrium Analysis

For each possible equilibrium $(s_a, s_b) \in \{$(C,N), (N,C), (N,N), (C,C)$\}$, we solve the problem by backward induction starting with the intermediaries' second period problems. Recall, however, that the intermediaries' second period problems (i.e., (P1) and (P7)) do not depend on the manufacturers' choice on buyback pricing regime. Therefore, first we characterize the Nash equilibrium between the intermediaries in the second period:

$$\begin{aligned}
q_{2n}^{a*} &= \frac{\gamma\theta_u - w_2^a + v^a}{2\gamma\theta_u} \\
q_{2n}^{b*} &= \frac{\gamma\theta_u - w_2^b + v^b}{2\gamma\theta_u} \\
\bar{q}_{2n}^{a*} &= \frac{1-\beta+\beta\bar{w}_2^b - \bar{w}_2^a}{2(1-\beta)(1+\beta)} - (1-\gamma\theta_u)(\bar{q}_{1n}^a - \bar{q}_v^a) \\
\bar{q}_{2n}^{b*} &= \frac{1-\beta+\beta\bar{w}_2^a - \bar{w}_2^b}{2(1-\beta)(1+\beta)} - (1-\gamma\theta_u)(\bar{q}_{1n}^b - \bar{q}_v^b) \\
q_{2p}^{a*} &= \frac{(1-\gamma\theta_u)w_2^a - v^a - (1-\theta_u)\gamma\theta_u q_{1n}^a}{2\gamma\theta_u(1-\gamma\theta_u)}
\end{aligned} \tag{28}$$

$$q_{2p}^{b*} = \frac{(1-\gamma\theta_u)w_2^b - v^b - (1-\theta_u)\gamma\theta_u q_{1n}^b}{2\gamma\theta_u(1-\gamma\theta_u)}$$

$$\bar{q}_v^{a*} = \max\left\{\beta(\bar{q}_{2n}^b + \bar{q}_{1n}^b - \bar{q}_v^b) + \bar{q}_{1n}^a + \bar{q}_{2n}^a - \frac{1-\gamma\theta_u - \bar{v}^a}{2(1-\gamma\theta_u)}, 0\right\} \tag{29}$$

$$\bar{q}_v^{b*} = \max\left\{\beta(\bar{q}_{2n}^a + \bar{q}_{1n}^a - \bar{q}_v^a) + \bar{q}_{1n}^b + \bar{q}_{2n}^b - \frac{1-\gamma\theta_u - \bar{v}^b}{2(1-\gamma\theta_u)}, 0\right\} \tag{30}$$

Since the manufacturers do not withhold any program cars, in the equilibrium we have $\bar{q}_v^{k*} = q_{2p}^{k*}$ for $k = \{a, b\}$. Therefore, for each possible choice of pricing regime, there are four possible outcomes with respect to buyback quantities as summarized in Figure 4. When $\bar{q}_v^{a*} = \bar{q}_v^{b*} = 0$, we call this zero-buyback outcome. When $\bar{q}_v^{a*} > 0$ and $\bar{q}_v^{b*} > 0$, we call this positive-buyback outcome. Finally when $\bar{q}_v^{a*} = 0$ and $\bar{q}_v^{b*} > 0$ (or when $\bar{q}_v^{a*} > 0$ and $\bar{q}_v^{b*} = 0$), only one manufacturer has a buyback program and we call this asymmetric-buyback outcome. As in §4, a manufacturer's choice of buyback price determines whether her buyback quantity would be positive or zero. From (29)-(30), we see that the buyback quantities can be positive when the associated buyback prices are sufficiently high.

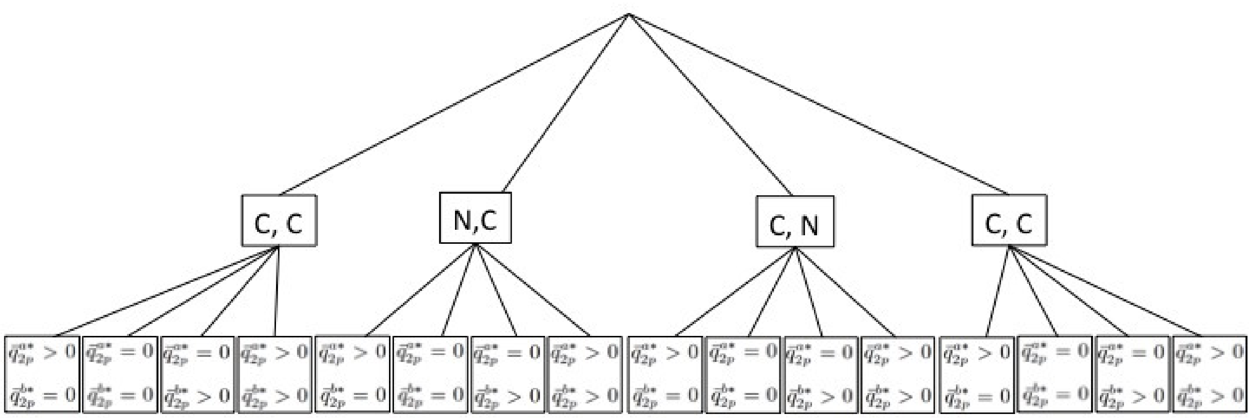


**Figure 4** **Possible equilibrium outcomes when there is competition**

**5.2.1 (C,C) Equilibrium** We now focus on the (C,C) equilibrium and aim to understand manufacturer profitability and buyback quantities in this setting, compared to the monopoly setting. Recall that in the absence of competition, our results show the optimal buyback quantity is always zero under precommitted buyback pricing (from Proposition 4). Next we provide a numerical example to demonstrate whether and how this result might change in the presence of competition. Let $\beta = 0.7$, $\theta_u = 0.4$, and $\gamma = 0.2$. Solving this problem by backward induction for each possible buyback outcome, we find the buyback quantity equilibirum as shown in Table 3.

| | | Manuf. b | |
|---|---|---|---|
| | | $\bar{q}_v^{b*}=0$ | $\bar{q}_v^{b*}>0$ |
| Manuf. a | $\bar{q}_v^{a*}=0$ | 0.3440,0.3440 | 0.3512,0.3446 |
| | $\bar{q}_v^{a*}>0$ | 0.3446 0.3512 | <u>0.3535</u>,<u>0.3535</u> |

**Table 3 Example: In a (C,C) equilibrium, manufacturer profits under each buyback quantity outcome. The buyback quantity equilibrium is underlined. The optimal buyback quantities are $\bar{q}_v^{a*}=\bar{q}_v^{b*}=0.0970$.**

This simple example demonstrates that when there is competition, a positive-buyback outcome may arise as the equilibrium when both manufacturers follow precommitted buyback pricing. Furthermore, in this example, the buyback prices are lower than the resale prices charged to the dealers (i.e., $\bar{v}^{a*}=\bar{v}^{b*}=0.1846<v^{a*}=v^{b*}=0.3783$). Note that this is in stark contrast to our findings under the monopoly setting. Recall that when a monopolist manufacturer chooses to precommit the buyback price, a positive-buyback outcome never arises in equilibrium because she has to offer a buyback price higher than the resale price charged to the dealer; and thus she loses money on every program car she buys back from the rental agency (i.e., $\bar{v}^*>v^*$ cf. Corollary 1). This example, however, demostrates that, amid competition the manufacturers can trade program cars profitably as they can set buyback prices lower than the resale prices (i.e., $\bar{v}^{k*}<v^{k*}$ for $k\in\{a,b\}$).

To explain why this happens, in what follows we conduct a comparative statics analysis. In explaining our results we will refer to the derivative of a decision variable with respect to a manufacturer's own prices or quantities as *self-partials* and with respect to the competitor's prices or quantities as *cross-partials*. Also, without loss of generality, we present the results from the view point of manufacturer $a$ and refer to manufacturer $b$ as the competitor. Next proposition states how second-period wholesale prices depend on the first-period quantities and buyback prices.

PROPOSITION 9. *In a (C,C) equilibrium, when the buyback quantities are positive (i.e., $\bar{q}_v^{a*}>0$ and $\bar{q}_v^{b*}>0$), manufacturer a's rental-car wholesale price in the second period depends on the buyback prices and first-period quantities as follows:* $\frac{\partial\bar{w}_2^{a*}}{\partial q_{1n}^a}>0$, $\frac{\partial\bar{w}_2^{a*}}{\partial\bar{q}_{1n}^a}>0$, $\frac{\partial\bar{w}_2^{a*}}{\partial\bar{v}^a}>0$, $\frac{\partial\bar{w}_2^{a*}}{\partial q_{1n}^b}>0$, $\frac{\partial\bar{w}_2^{a*}}{\partial\bar{q}_{1n}^b}>0$, *and* $\frac{\partial\bar{w}_2^{a*}}{\partial\bar{v}^b}<0$.

Recall that, in the absence of competition, the rental-car wholesale price announced by a manufacturer (see equation (21)) increases in first-period quantities and the buyback price. Comparing this with the findings in Proposition 9, we observe that the self-partials directionally remain the same. However, a manufacturer's rental-car wholesale price now also depends on her competitor's first-period quantities and buyback price. While the effect of competitor's quantities is also directionally the same as the effect of manufacturer's own quantities, the effect of competitor's buyback price is the opposite. Specifically, as the competitor increases her buyback price (e.g., $\bar{v}^b$ increases), the manufacturer's rental-car wholesale price ($\bar{w}_2^{a*}$) decreases. That is because once manufacturer $b$

increases her buyback price, the rental agency would return more program cars to manufacturer $b$. As indicated by (29), this causes the rental agency to return fewer cars to manufacturer $a$. As a result, manufacturer $a$ has to offer a lower wholesale price in order to incentivize the purchase of new cars, given that the rental agency already has more program cars of manufacturer $a$. Next we identify how first-period quantities change with wholesale and buyback prices.

| | | **(C,C) Equilibrium** | | | | | | **(N,N) Equilibrium** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\bar{w}_1^a$ | $\bar{w}_1^b$ | $w_1^a$ | $w_1^b$ | $\bar{v}^a$ | $\bar{v}^b$ | $\bar{w}_1^a$ | $\bar{w}_1^b$ | $w_1^a$ | $w_1^b$ | $\bar{v}^a$ | $\bar{v}^b$ |
| $\bar{q}_v^{a*} > 0,\ \bar{q}_v^{b*} > 0$ | $q_{1n}^a$ | + | - | - | + | - | + | + | - | - | + | NA | NA |
| | $q_{1n}^b$ | - | + | + | - | + | - | - | + | + | - | NA | NA |
| | $\bar{q}_{1n}^a$ | - | + | - | + | + | - | - | + | + | + | NA | NA |
| | $\bar{q}_{1n}^b$ | + | - | + | - | - | + | + | - | + | + | NA | NA |
| $\bar{q}_v^{a*} > 0,\ \bar{q}_v^{b*} = 0$ | $q_{1n}^a$ | + | - | - | 0 | - | 0 | + | + | - | 0 | NA | NA |
| | $q_{1n}^b$ | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | - | NA | NA |
| | $\bar{q}_{1n}^a$ | - | + | - | 0 | + | 0 | - | + | + | 0 | NA | NA |
| | $\bar{q}_{1n}^b$ | + | - | + | 0 | - | 0 | + | - | + | 0 | NA | NA |
| $\bar{q}_v^{a*} = 0,\ \bar{q}_v^{b*} = 0$ | $q_{1n}^a$ | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | - | 0 | NA | NA |
| | $q_{1n}^b$ | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | - | NA | NA |
| | $\bar{q}_{1n}^a$ | - | + | 0 | 0 | 0 | 0 | - | + | 0 | 0 | NA | NA |
| | $\bar{q}_{1n}^b$ | + | - | 0 | 0 | 0 | 0 | + | - | 0 | 0 | NA | NA |
| $\bar{q}_v^{a*} = 0,\ \bar{q}_v^{b*} > 0$ | $q_{1n}^a$ | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | - | 0 | NA | NA |
| | $q_{1n}^b$ | - | + | 0 | - | 0 | - | + | + | 0 | - | NA | NA |
| | $\bar{q}_{1n}^a$ | - | + | 0 | + | 0 | - | - | + | 0 | + | NA | NA |
| | $\bar{q}_{1n}^b$ | + | - | 0 | - | 0 | + | + | - | 0 | + | NA | NA |

**Table 4** **Direction of change in first-period quantities with respect to prices in the (C,C) and (N,N) equilbria**

Proposition 10. *In a (C,C) equilibrium, when the buyback quantities are positive (i.e., $\bar{q}_v^{a*} > 0$ and $\bar{q}_v^{b*} > 0$), in the first period*

1. *Rental agency's order quantity depends on the buyback and wholesale prices as follows:* $\frac{\partial \bar{q}_{1n}^{a*}}{\partial \bar{v}^a} > 0$, $\frac{\partial \bar{q}_{1n}^{a*}}{\partial \bar{v}^b} < 0$, $\frac{\partial \bar{q}_{1n}^{a*}}{\partial \bar{w}_1^a} < 0$, $\frac{\partial \bar{q}_{1n}^{a*}}{\partial w_1^a} < 0$, $\frac{\partial \bar{q}_{1n}^{a*}}{\partial \bar{w}_1^b} > 0$, *and* $\frac{\partial \bar{q}_{1n}^{a*}}{\partial w_1^b} > 0$.
2. *Dealer a's order quantity depends on the buyback and wholesale prices as follows:* $\frac{\partial q_{1n}^{a*}}{\partial \bar{v}^a} < 0$, $\frac{\partial q_{1n}^{a*}}{\partial \bar{v}^b} > 0$, $\frac{\partial q_{1n}^{a*}}{\partial \bar{w}_1^a} > 0$, $\frac{\partial q_{1n}^{a*}}{\partial w_1^a} < 0$, $\frac{\partial q_{1n}^{a*}}{\partial \bar{w}_1^b} < 0$, *and* $\frac{\partial q_{1n}^{a*}}{\partial w_1^b} > 0$.

*The complete list of the derivaties is given in Table 4.*

Comparing Proposition 10 with Proposition 1, we observe that self-partials directionally remain the same whether manufacturer competition exists or not. The difference between the monopoly and competition settings comes from the cross-partials under competition. In particular, the rental agency orders more from manufacturer $a$ ($b$) in the first period as manufacturer $b$ ($a$) decreases her buyback price (i.e., $\frac{\partial \bar{q}_{1n}^{a*}}{\partial \bar{v}^b} < 0$ and $\frac{\partial \bar{q}_{1n}^{b*}}{\partial \bar{v}^a} < 0$). That is because, ceteris paribus, as one manufacturer lowers her buyback price, the rental agency finds purchasing the competitor's cars more attractive. Therefore, this gives both manufacturers an opportunity to lower their buyback prices at the same time without hurting their first-period rental car sales significantly. In other words, the manufacturers' buyback prices act as *strategic complements*. Because of this strategic complementarity, the manufacturers now can set buyback prices lower than the resale prices charged to the dealers and make profit on each program car repurchased from the rental agency (recall that as discussed in §4.1, this is not possible for a monopolist manufacturer). Finally, we also show that the magnitude of increase in the first-period order quantity ($\bar{q}_{1n}^{a*}$) as the competitor's buyback price ($\bar{v}^b$) decreases is larger when $\beta$ is high (i.e., $\frac{\partial^2 \bar{q}_{1n}^{a*}}{\partial \bar{v}^b \partial \beta} < 0$). Put differently, the strategic complementarity of the buyback prices is stronger when $\beta$ is higher.

Our discussion so far shows that the manufacturers can achieve positive-buyback outcome under precommitted pricing. This alone, however, does not answer if/when manufacturers would prefer choosing precommitment over postponement. To this end, we also solve the problem for the other possible choices of pricing regime (i.e., (N,C), (C,N), and (N,N)). Analysis and results similar to those in this section are available in the appendix but omitted in the paper for brevity. Next, using the same example we provided in Table 3, we demostrate that (C,C) pricing-regime with positive-buyback outcome can arise in the equilibrium, as shown in Table 5. In this specific example, the buyback outcome is always positive in all possible pricing regimes, i.e., for (C,C), (C,N), and (N,N).

| | | Manuf. b: C | Manuf. b: N |
|---|---|---|---|
| Manuf. a | C | <u>0.3535</u>, <u>0.3535</u> | 0.3446, 0.3526 |
| | N | 0.3526, 0.3446 | 0.3445,0.3445 |

**Table 5 Manufacturer profits under each possible buyback pricing regime for $\beta = 0.7$, $\theta_u = 0.4$, and $\gamma = 0.2$. The equilibrium profits are underlined.**

It is worth noting that, the optimal buyback quantity under (N,N) pricing regime is $\bar{q}_v^{a*} = \bar{q}_v^{b*} = 0.0644$ which is lower than the optimal buyback quantity under (C,C) pricing regime (recall that it is $\bar{q}_v^{a*} = \bar{q}_v^{b*} = 0.0970$ from Table 3). Therefore, precommitting buyback prices not only increases the manufacturers' profits but also gives them a larger buyback program.

To sum up, in the presence of competition, due to strategic complementarity of the buyback prices, the manufacturers choose low buyback prices thereby making precommited buyback pricing an attractive option. Next we provide an extensive numerical study to identify the conditions under which precommitted buyback pricing arises in equilibrium under competition.

### 5.3 The Equilibrium Choices of Buyback Pricing Regime: Numerical Analysis

The complete characterization of the subgame perfect equilibrium of buyback pricing regimes is analytically intractable. In order to find the equilibrium, we conduct a thorough numerical analysis by scanning the entire feasible regions of these problem parameters. We use $\beta \in \{0.1, ..., 0.95\}$, $\gamma = \{0.05, 0.1, ..., 1\}$, and $\theta_u = \{0.05, 0.1, ..., 1\}$ and therefore we solve 19x20x20=7600 instances of the problem for each one of the four possible pricing-regime outcomes. Given that all three parameters are between 0 and 1, our numerical analysis covers the entire parameter space and thus is comprehensive. Based on this extensive numerical study, we next selectively present most predominant equilibrium patterns and make some observations regarding when each equilibrium may arise.

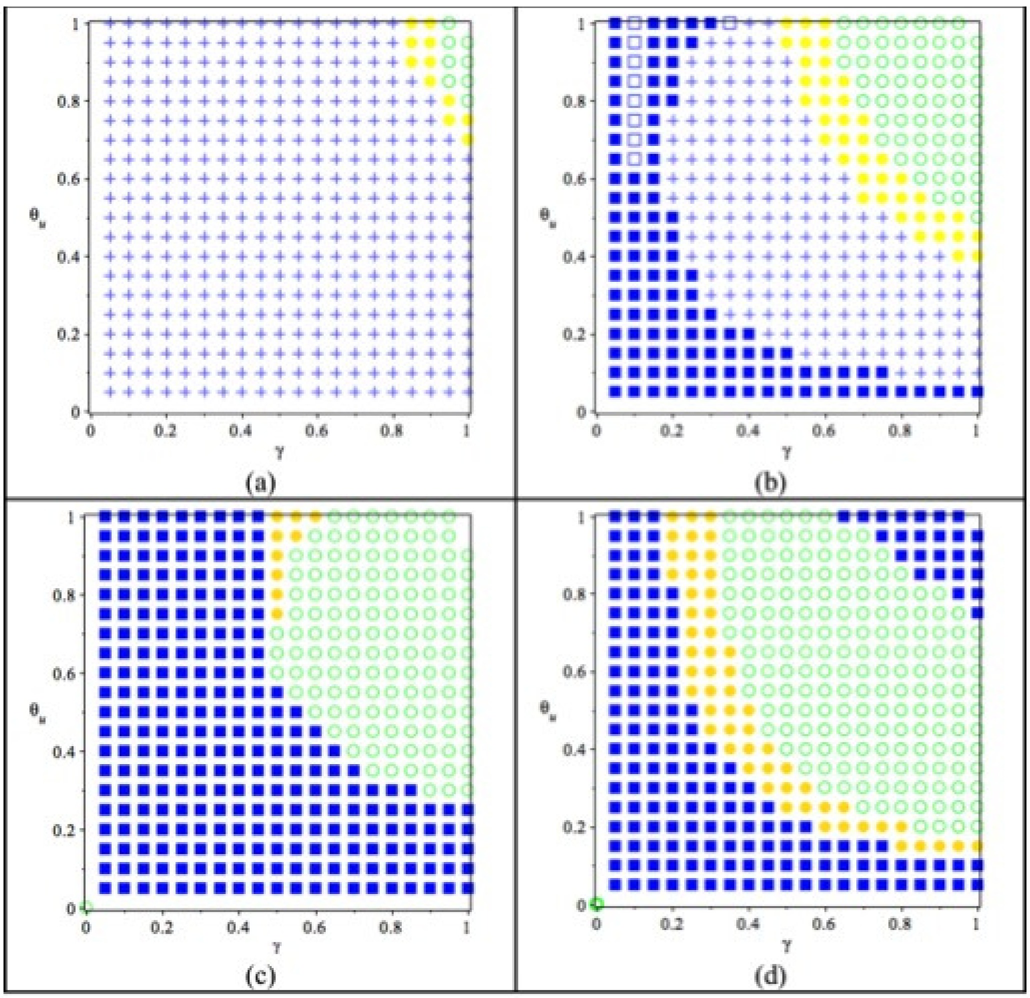


**Figure 5** **Buyback pricing equilibrium when (a)** $\beta = 0.4$**, (b)** $\beta = 0.6$**, (c)** $\beta = 0.8$**, and (d)** $\beta = 0.9$**; where + denotes (N,N) equilibrium, ○ denotes multiple equilibria with zero-buyback quantities for both manufacturers, ● denotes (C,N)/(N,C) equilibrium, ■ denotes (C,C) equilibrium, and □ denotes (N,N) equilibrium as a prisoner's dilemma.**

When the substitutability of manufacturers' cars is small ($\beta \leq 0.20$), we observe that (N,N) pricing regime with positive-buyback outcome arises in equilibrium (for all values of $\gamma$ and $\theta_u$).

Note that when $\beta$ is low, the competition in the rental market is weak and the manufacturers behave almost like monopolists. Therefore, we observe the same equilibrium behavior as that in the monopoly model, which is stated in the following observation:

OBSERVATION 1: *When $\beta$ is sufficiently low, both manufacturers choose postponed buyback pricing in equilibrium (i.e., (N,N) equilibrium arises).*

For moderate values of $\beta$ (i.e., $0.2 \leq \beta \leq 0.45$), we still observe (N,N) equilibrium with positive buyback quantities, but only when the depreciation of cars is sufficiently low. For example, Figure 5(a) depicts the equilibrium solution with respect to $\gamma$ and $\theta_u$ for $\beta = 0.40$ (see crosses for (N,N) equilibrium). When the depreciation of cars is relatively higher (see solid circles in Figure 5(a)) we observe (C,N) (or, equivalently (N,C)) equilibrium with positive-buyback outcome only for the manufacturer who follows postponement (N). Therefore, one manufacturer can offer a high enough buyback price to induce positive-buyback outcome, but only when she follows postponed buyback pricing. Finally, we observe that when cars depreciate significantly (i.e., when both $\gamma$ and $\theta_u$ are high), regardless of the choice of pricing regime, the buyback quantity is zero for both manufacturers (see empty circles in Figure 5(a)), and multiple equilibria arise (i.e., $(s_a^*, s_b^*)$ where $s_a^*, s_b^* \in \{N, C\}$). In this case, due to moderate competition in the rental market and significant depreciation of cars, the manufacturers cannot reduce buyback prices enough to turn the buyback program into a profitable option.[5]

For higher values of $\beta$ (i.e., $\beta \geq 0.5$), we observe that both parties may choose precommitted buyback pricing (C,C) yielding positive buyback quantities (see solid squares in Figure 5(b)-(d)). This is consistent with the discussion following Proposition 10: When $\beta$ is high, the strategic complementarity of the buyback prices is strong, inducing a (C,C) equilibrium. This is summarized in the following observation:

OBSERVATION 2: *When $\beta$ is sufficiently large, an equilibrium may arise in which both manufacturers choose precommitted buyback pricing with positive buyback quantities (i.e., (C,C) equilibrium).*

Comparing Figure 5(b)-(d), we observe that the emergence of (C,C) equilibrium also depends on the depreciation rates. For moderate values of $\beta$ (i.e., for $0.5 \leq \beta \leq 0.80$) we observe that (C,C) equilibrium arises only when the depreciation of cars is small (see solid squares in the lower left corner of Figures 5(b)-(c)). When used cars do not depreciate much, new cars would face stiffer competition in the sales market. Therefore, the manufacturers find it attractive to sell program

[5] Deviating unilaterally by offering a high buyback price to induce a positive-buyback outcome would hurt the manufacturer profit, because (i) trading depreciated program cars is not attractive and (ii) a higher buyback price leads to a higher rental-car wholesale price in the second period; causing the rental agency to buy from the competitor (who offers lower wholesale price) to replace the cars returned to the deviating manufacturer.

cars –which are also not much depreciated– at a cheaper price. If they precommit buyback prices, they can sell more rental cars in the first period, buy them back at reasonable prices in the second period to sell them through the dealers. When the competition in the rental market is more intense (i.e., for $\beta \geq 0.85$) we observe that (C,C) equilibrium arises for low levels of depreciation as before (see solid squares in the lower left corner of Figure 5(d)). In addition, it also arises for high levels of depreciation (see solid squares in the upper right corner of Figure 5(d)).

It is worth noting that, whenever (C,C) pricing regime gives positive-buyback outcome (for both manufacturers) it emerges as equilibrium, except for rare cases. For example, there are instances where (N,N) pricing regime yields lower manufacturer profits than (C,C) pricing regime but still arises in equilibrium, due to prisoner's dilemma (see empty squares in Figure 5(b)).

Finally, we identify an asymetric equilibrium in which commitment yields positive-buyback quantity: When the competition is intense (i.e., $\beta \geq 0.80$) we observe (N,C) (or equivalently (C,N)) equilibrium with positive-buyback outcome only for the manufacturer who follows precommitment (C) (see solid dots in Figure 5(c)-(d)). Here, a manufacturer may reduce her buyback price sufficiently to make profit on the transaction of the program cars even when her competitor achieves zero-buyback quantity. Therefore, we make the following observation:

OBSERVATION 3: *When $\beta$ is sufficiently large, an equilibrium may arise in which one manufacturer chooses precommitted buyback pricing with positive-buyback quantity while the other manufacturer chooses postponed buyback pricing with zero-buyback quantity.*

To sum up, when the manufacturer competition is weak in the rental market, we observe the (N,N) equilibrium. However, when the manufacturer competition is sufficiently intense in the rental market, we may observe the (C,C) equilibrium with positive buyback outcomes depending on how products depreciate overtime.

## 6. Conclusions

In this paper, we study manufacturers' buyback pricing of durable goods in dual distribution channels consisting of a dealer and a rental agency. The manufacturers buyback used rental products from rental agencies and redistribute them through dealers. Our characterization of the strategic interactions between the manufacturers and the intermediaries generates useful insights about managing dual distribution channels. Specifically, we find that precommitting buyback prices to rental agencies at the time of initial sales may hurt a monopolist manufacturer's profit, compared to postponing the pricing decision to the time of repurchase. Our analysis suggests that precommitting buyback price puts the manufacturer in a disadvantageous position – the intermediaries adjust their first-period ordering decisions based on the buyback price. This causes the manufacturer to face a challenging tradeoff when managing the intermediaries: when she offers an attractive (i.e., high) buyback price to the rental agency, she would have to offer an attractive (i.e., low) wholesale price to the dealer. This tradeoff hurts the manufacturer's profit because it makes the buyback program expensive – the manufacturer has to offer a sufficiently high buyback price to induce a positive buyback outcome. Under postponed buyback pricing, however, since the manufacturer announces the buyback price after observing intermediaries' first-period ordering decisions, this allows her to avoid the aforementioned tradeoff and set a low buyback price, which still induces a positive buyback outcome. Therefore, under postponed buyback pricing the manufacturer can profitably repurchase program cars from rental agencies and redistribute them through dealers.

Extending the model to competition between two manufacturers selling to the same rental agency, we show that precommitted buyback pricing with positive buyback quantities arises in equilibrium when competition is sufficiently intense. Specifically, we find that the manufacturers' buyback prices act as strategic complements. Therefore, competition gives manufacturers an opportunity to decrease their buyback prices while still inducing positive buyback quantities. Our findings, therefore, provide a plausible explanation to the prevalence of buyback programs with guaranteed pricing in the automobile industry.

Finally, our paper is not without its limitations. First, we consider a deterministic demand model. Our model can be extended by incorporating uncertainty in the depreciation rates to capture the reality that the residual value of used cars is stochastic and to introduce the mixed demand uncertainty for new, program, and used cars. We conjecture that adding uncertainty may make manufacturers more likely to precommit buyback prices in the presence of competition. This is because assurance of buyback prices could induce rental agencies to buy more new cars. Second, we focus on manufacturer competition in the rental market because buyback programs have stronger connection to the rental market. Nevertheless, our model can be extended by incorporating the sales market competition. These are interesting research questions to be answered in future research.

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## Appendix A: Characterization of Monopoly Solution

We solve the problem by backward induction. First, we write the Lagrangian of the rental agency's problem as $L_R = \bar{\pi}_2 + b_R \bar{q}_v$ where $b_R$ is the Lagrangian multiplier. Then the first order conditions (FOCs) are

$\frac{\partial L_R}{\bar{q}_{2n}} = (1-\gamma\theta_u)(1-\bar{q}_{2n}-2\bar{q}_{1n}+2\bar{q}_v)+\gamma\theta_u(1-\bar{q}_{2n})-\bar{w}_2-\bar{q}_{2n}=0,$

$\frac{\partial L_R}{\bar{q}_v} = \bar{v}-(1-\gamma\theta_u)(1-2\bar{q}_{2n}-2\bar{q}_{1n}+2\bar{q}_v)+b_R=0,$

$\frac{\partial \pi_2}{q_{2n}} = 1-q_{1n}(1-\theta_u)-2q_{2n}-w_2-2q_{2p}(1-\gamma\theta_u)=0,$

$\frac{\partial \pi_2}{q_{2p}} = (1-\gamma\theta_u)(1-2q_{2p}-2q_{2n})-q_{1n}(1-\theta_u)-v+b_D=0.$

In addition, the complementary slackeness condition $b_R\bar{q}_v = 0$ as well as feasibilily conditions $\bar{q}_v \geq 0$ and $b_R \geq 0$ should hold. Solving all optimality conditions simultaneously, we find two possible equilibrium solutions:

*-Solution 1:* $b_R^* = -2\gamma\theta_u\bar{q}_{1n}(1-\gamma\theta_u)+(1-\gamma\theta_u)\bar{w}_2-\bar{v}$, $q_{2n}^* = \frac{\gamma\theta_u-w_2+v}{2\gamma\theta_u}$, $\bar{q}_{2n}^* = \frac{1-\bar{w}_2-2(1-\gamma\theta_u)\bar{q}_{1n}}{2}$, $q_{2p}^* = \frac{-(1-\theta_u)\gamma\theta_u q_{1n}+(1-\gamma\theta_u)w_2-v}{2\gamma\theta_u(1-\gamma\theta_u)}$, $\bar{q}_v^* = 0$,

*-Solution 2:* $b_R^* = 0$, $q_{2n}^* = \frac{\gamma\theta_u-w_2+v}{\gamma\theta_u}$, $\bar{q}_{2n}^* = \frac{\gamma\theta_u-\bar{w}_2+\bar{v}}{2\gamma\theta_u}$, $q_{2p}^* = \frac{-(1-\theta_u)\gamma\theta_u q_{1n}+(1-\gamma\theta_u)w_2-v}{2\gamma\theta_u(1-\gamma\theta_u)}$, $\bar{q}_v^* = \frac{2\gamma\theta_u\bar{q}_{1n}(1-\gamma\theta_u)-(1-\gamma\theta_u)\bar{w}_2+\bar{v}}{2\gamma\theta_u(1-\gamma\theta_u)}$.

Recall that the resale price $v_p^*$ comes from the equilibrium $\bar{q}_v^* = q_{2p}^*$. Therefore, under solution 1 we get $v_p^* = (1-\gamma\theta_u)w_2-\gamma\theta_u(1-\theta_u)q_{1n}$ from $\bar{q}_v^* = q_{2p}^* = 0$; whereas under solution 2 we get $v_p^* = (1-\gamma\theta_u)(w_2+\bar{w}_2)-\gamma\theta_u(1-\theta_u)q_{1n}-2\gamma\theta_u(1-\gamma\theta_u)\bar{q}_{1n}-\bar{v}$ from $\bar{q}_v^* = q_{2p}^* > 0$. Accordingly, solution 1 results in zero-buyback outcome, i.e., $\bar{q}_v^* = q_{2p}^* = 0$; and solution 2 results in positive-buyback outcome, i.e., $\bar{q}_v^* = q_{2p}^* > 0$. For each solution, we continue with standard application of backward induction. For the sake of brevity we do not present details of each intermediate step (full analysis is available from the authors upon request). Below we first present the final solutions. We verify that all the expressions for optimal values of variables and objective functions in the final solutions are nonnegative in the region of interest (i.e., when $\gamma \geq 0$ and $\theta_u \geq 0$)

### A.1 Zero-Buyback Outcome (For both precommitted and postponed buyback pricing):

Recall that buyback pricing regime becomes irrelevant under zero-buyback outcome. Therefore, using solution 1 ($\bar{q}_v^* = q_{2p}^* = 0$) in backward induction for both precommitment and postponement regimes gives the following:

$$\bar{w}_1^* = \frac{11\gamma^3\theta_u^3-25\gamma^2\theta_u^2-19\gamma\theta_u+49}{16(3-\gamma^2\theta_u^2)} \tag{31}$$

$$w_1^* = \frac{45\theta_u^3-79\theta_u^2-217\theta_u+379}{32(13-3\theta_u^2-2\theta_u)} \tag{32}$$

$$\bar{q}_{1n}^* = \frac{5-\gamma\theta_u}{8(3-\gamma^2\theta_u^2)} \tag{33}$$

$$q_{1n}^* = \frac{11-3\theta_u}{4(13-3\theta_u^2-2\theta_u)} \tag{34}$$

$$\bar{w}_2^* = \frac{7-5\gamma^2\theta_u^2+6\gamma\theta_u}{8(3-\gamma^2\theta_u^2)} \tag{35}$$

$$w_2^* = \frac{41-15\theta_u^2+6\theta_u}{8(13-3\theta_u^2-2\theta_u)} \tag{36}$$

$$\bar{q}_{2n}^* = \frac{7-5\gamma^2\theta_u^2+6\gamma\theta_u}{16(3-\gamma^2\theta_u^2)} \tag{37}$$

$$q_{2n}^* = \frac{41 - 15\theta_u^2 + 6\theta_u}{16(13 - 3\theta_u^2 - 2\theta_u)} \tag{38}$$

$$\bar{\pi}_1^* = \frac{15\gamma^4\theta_u^4 + 8\gamma^3\theta_u^3 - 94\gamma^2\theta_u^2 - 120\gamma\theta_u + 319}{256(3 - \gamma^2\theta_u^2)^2} \tag{39}$$

$$\pi_1^* = \frac{99\theta_u^4 + 468\theta_u^3 - 1150\theta_u^2 - 3340\theta_u + 5971}{256(13 - 3\theta_u^2 - 2\theta_u)^2} \tag{40}$$

$$\Pi_1^* = \frac{18\gamma^2\theta_u^4 + 48\gamma^2\theta_u^3 - 158\gamma^2\theta_u^2 + 15\gamma\theta_u^3 + 10\gamma\theta_u^2 - 65\gamma\theta_u - 96\theta_u^2 - 172\theta_u + 656}{32((3 - \gamma^2\theta_u^2)(13 - 3\theta_u^2 - 2\theta_u))} \tag{41}$$

Furthermore, substituting the optimal values of variables in $b_R^*$ of solution 1, we find $b_R^* = \frac{3\gamma^3\theta_u^3 + \gamma^2\theta_u^2 - 11\gamma\theta_u + (8\gamma^2\theta_u^2 - 24)\bar{v} + 7}{8(3 - \gamma^2\theta_u^2)}$. Note that $b_R^* \geq 0$ can be written as $\bar{v} \leq \bar{v}_T$ where

$$\bar{v}_T = \frac{(3\gamma\theta_u + 7)(1 - \gamma\theta_u)^2}{8(3 - \gamma^2\theta_u^2)}. \tag{42}$$

This concludes the solution for zero buyback outcome. Positive buyback outcome, however, needs to be analyzed separately under precommitted and postponed buyback pricing. We present our findings next:

## A.2 Precommitted Buyback Pricing with Positive-Buyback Outcome:

Using solution 2 ($\bar{q}_v^* = q_{2p}^* > 0$) in backward induction for precommitted buyback pricing, we find:

$$\bar{v}^* = \frac{1 - \gamma\theta_u}{A} \begin{pmatrix} -3328 + 4\gamma^6\theta_u^8 + (24\gamma^6 - 45\gamma^5)\theta_u^7 - (44\gamma^6 + 246\gamma^5 - 207\gamma^4)\theta_u^6 + (499\gamma^5 + 1434\gamma^4 - 726\gamma^3)\theta_u^5 + (-3001\gamma^4 - 4140\gamma^3 + 1636\gamma^2)\theta_u^4 \\ +(9394\gamma^3 + 5672\gamma^2 - 1832\gamma)\theta_u^3 + (-15020\gamma^2 - 3312\gamma + 768)\theta_u^2 + (11544\gamma + 512)\theta_u \end{pmatrix} \tag{43}$$

$$v^* = \frac{1 - \gamma\theta_u}{A} \begin{pmatrix} 2624 - 9\gamma^6\theta_u^8 - (66\gamma^6 - 95\gamma^5)\theta_u^7 + (123\gamma^6 + 526\gamma^5 - 439\gamma^4)\theta_u^6 - (1053\gamma^5 + 2030\gamma^4 - 1354\gamma^3)\theta_u^5 + (4501\gamma^4 + 4088\gamma^3 - 2565\gamma^2)\theta_u^4 \\ -(11026\gamma^3 + 3838\gamma^2 - 2512\gamma)\theta_u^3 + (14947\gamma^2 + 992\gamma - 960)\theta_u^2 - (10160\gamma - 384)\theta_u \end{pmatrix} \tag{44}$$

$$\bar{w}_1^* = \frac{1}{2A} \begin{pmatrix} -6656 - (32\gamma^8 + 10\gamma^7)\theta_u^9 + (64\gamma^8 + 404\gamma^7 + 62\gamma^6)\theta_u^8 - (762\gamma^7 + 1944\gamma^6 + 222\gamma^5)\theta_u^7 + (3578\gamma^6 + 4392\gamma^5 + 907\gamma^4)\theta_u^6 - (7722\gamma^5 + 3986\gamma^4 + 2850\gamma^3)\theta_u^5 \\ +(4583\gamma^4 - 1688\gamma^3 + 5001\gamma^2)\theta_u^4 + (12554\gamma^3 + 6486\gamma^2 - 4408\gamma)\theta_u^3 - (28287\gamma^2 + 4688\gamma - 1536)\theta_u^2 + (22664\gamma + 1024)\theta_u \end{pmatrix}$$

$$w_1^* = \frac{1}{4(\gamma\theta_u - 2)A} \begin{pmatrix} 24256 - (24\gamma^8 + 15\gamma^7)\theta_u^{10} + (104\gamma^8 + 305\gamma^7 + 144\gamma^6)\theta_u^9 - (112\gamma^8 + 1213\gamma^7 + 1552\gamma^6 + 675\gamma^5)\theta_u^8 + (1307\gamma^7 + 5696\gamma^6 + 3793\gamma^5 + 2362\gamma^4)\theta_u^7 \\ -(6144\gamma^6 + 11257\gamma^5 + 4918\gamma^4 + 6097\gamma^3)\theta_u^6 + (11979\gamma^5 - 570\gamma^4 + 5471\gamma^3 - 9781\gamma^2)\theta_u^5 + (3350\gamma^4 + 42077\gamma^3 - 9367\gamma^2 - 8352\gamma)\theta_u^4 \\ -(58219\gamma^3 + 73825\gamma^2 - 11232\gamma - 2880)\theta_u^3 + (106051\gamma^2 + 53024\gamma + 5056)\theta_u^2 - (82528\gamma + 13888)\theta_u) \end{pmatrix}$$

$$\bar{q}_{1n}^* = \frac{1}{4A} \begin{pmatrix} -3328 + 5\gamma^6\theta_u^8 + (74\gamma^6 - 52\gamma^5)\theta_u^7 - (143\gamma^6 + 756\gamma^5 - 302\gamma^4)\theta_u^6 + (1448\gamma^5 + 3032\gamma^4 - 1032\gamma^3)\theta_u^5 + (-6086\gamma^4 - 6132\gamma^3 + 1977\gamma^2)\theta_u^4 \\ +(13692\gamma^3 + 6502\gamma^2 - 1952\gamma)\theta_u^3 + (-17439\gamma^2 - 3264\gamma + 768)\theta_u^2 + (11872\gamma + 512)\theta_u \end{pmatrix}$$

$$q_{1n}^* = \frac{(1 - \gamma\theta_u)}{A} \left( -704 + \gamma^6\theta_u^7 + (3\gamma^6 - 14\gamma^5)\theta_u^6 + (-22\gamma^5 + 72\gamma^4)\theta_u^5 + (44\gamma^4 - 226\gamma^3)\theta_u^4 + (166\gamma^3 + 443\gamma^2)\theta_u^3 + (-883\gamma^2 - 464\gamma)\theta_u^2 + (1360\gamma + 192)\theta_u \right)$$

$$\bar{w}_2^* = \frac{1}{2A} \begin{pmatrix} -3328 + 4\gamma^7\theta_u^9 + (56\gamma^7 - 33\gamma^6)\theta_u^8 + (-108\gamma^7 - 486\gamma^6 + 150\gamma^5)\theta_u^7 + (935\gamma^6 + 1596\gamma^5 - 397\gamma^4)\theta_u^6 + (-3250\gamma^5 - 1998\gamma^4 + 284\gamma^3)\theta_u^5 \\ +(4827\gamma^4 - 480\gamma^3 + 784\gamma^2)\theta_u^4 + (-172\gamma^3 + 3376\gamma^2 - 1544\gamma)\theta_u^3 + (-8416\gamma^2 - 2608\gamma + 768)\theta_u^2 + (9528\gamma + 512)\theta_u \end{pmatrix}$$

$$w_2^* = \frac{1}{2A} \begin{pmatrix} -2624 - \gamma^7\theta_u^9 + (14\gamma^7 + 22\gamma^6)\theta_u^8 - (29\gamma^7 + 164\gamma^6 + 132\gamma^5)\theta_u^7 + (302\gamma^6 + 720\gamma^5 + 463\gamma^4)\theta_u^6 - (1196\gamma^5 + 1454\gamma^4 + 1273\gamma^3)\theta_u^5 \\ +(1695\gamma^4 + 1406\gamma^3 + 2393\gamma^2)\theta_u^4 + (1387\gamma^3 - 778\gamma^2 - 2416\gamma)\theta_u^3 - (6959\gamma^2 - 608\gamma - 960)\theta_u^2 + (7440\gamma - 384)\theta_u \end{pmatrix}$$

$$\bar{q}_{2n}^* = \frac{1}{2A} \begin{pmatrix} 328 + (16\gamma^6 + 3\gamma^5)\theta_u^7 - (32\gamma^6 + 182\gamma^5 - \gamma^4)\theta_u^6 + (339\gamma^5 + 714\gamma^4 - 34\gamma^3)\theta_u^5 - (1323\gamma^4 + 1244\gamma^3 + 7\gamma^2)\theta_u^4 + (2302\gamma^3 + 986\gamma^2 + 149\gamma)\theta_u^3 \\ -(1571\gamma^2 + 322\gamma + 120)\theta_u^2 - (51\gamma - 48)\theta_u)(\gamma\theta_u - 2) \end{pmatrix}$$

$$q_{2n}^* = \frac{1}{2A} \begin{pmatrix} 328 + (16\gamma^6 + 3\gamma^5)\theta_u^7 + (-32\gamma^6 - 182\gamma^5 + \gamma^4)\theta_u^6 + (339\gamma^5 + 714\gamma^4 - 34\gamma^3)\theta_u^5 + (-1323\gamma^4 - 1244\gamma^3 - 7\gamma^2)\theta_u^4 + (2302\gamma^3 + 986\gamma^2 + 149\gamma)\theta_u^3 \\ -(1571\gamma^2 + 322\gamma + 120)\theta_u^2 + (-51\gamma + 48)\theta_u)(\gamma\theta_u - 2) \end{pmatrix}$$

$$\bar{q}_v^* = q_{2p}^* = \frac{1}{4A} \begin{pmatrix} -1312 + \gamma^6\theta_u^8 + (18\gamma^6 - 15\gamma^5)\theta_u^7 + (-35\gamma^6 - 246\gamma^5 + 107\gamma^4)\theta_u^6 + (469\gamma^5 + 1190\gamma^4 - 428\gamma^3)\theta_u^5 + (-2337\gamma^4 - 2700\gamma^3 + 967\gamma^2)\theta_u^4 \\ +(5864\gamma^3 + 2842\gamma^2 - 1100\gamma)\theta_u^3 + (-7873\gamma^2 - 968\gamma + 480)\theta_u^2 + (5268\gamma - 192)\theta_u \end{pmatrix}$$

$$\Pi_1^* = \frac{1}{8(2 - \gamma\theta_u)A} \begin{pmatrix} -27712 + \gamma^8\theta_u^{10} + (6\gamma^8 - 22\gamma^7)\theta_u^9 + (9\gamma^8 - 124\gamma^7 + 185\gamma^6)\theta_u^8 - (14\gamma^7 - 1166\gamma^6 + 882\gamma^5)\theta_u^7 - (1031\gamma^6 + 6828\gamma^5 - 3111\gamma^4)\theta_u^6 \\ +(10622\gamma^5 + 23898\gamma^4 - 8026\gamma^3)\theta_u^5 - (47121\gamma^4 + 48500\gamma^3 - 13025\gamma^2)\theta_u^4 + (111502\gamma^3 + 55278\gamma^2 - 11376\gamma)\theta_u^3 \\ -(146191\gamma^2 + 32288\gamma - 4032)\theta_u^2 + (99984\gamma + 7296)\theta_u \end{pmatrix} \tag{45}$$

where $A = -3328 + (16\gamma^7 + 7\gamma^6)\theta_u^8 - (32\gamma^7 + 174\gamma^6 + 46\gamma^5)\theta_u^7 + (327\gamma^6 + 700\gamma^5 + 165\gamma^4)\theta_u^6 - (1262\gamma^5 + 1034\gamma^4 + 604\gamma^3)\theta_u^5 + (1573\gamma^4 - 312\gamma^3 + 1486\gamma^2)\theta_u^4 + (2436\gamma^3 + 2372\gamma^2 - 1760\gamma)\theta_u^3 - (9202\gamma^2 + 2112\gamma - 768)\theta_u^2 +$

$(9504\gamma+512)\theta_u$. The expressions of intermediaries' profit functions are tedious and omitted for brevity; but available from the authors upon request.

### A.3 Postponed Buyback Pricing with Positive-Buyback Outcome:

Using solution 2 ($\bar{q}_v^* = q_{2p}^* > 0$) in backward induction for precommitted buyback pricing, we find:

$$\bar{v}^* = \frac{(1-\gamma\theta_u)(364-8\gamma^3\theta_u^5+(3\gamma^2-48\gamma^3)\theta_u^4+(120\gamma^3-34\gamma^2+71\gamma)\theta_u^3+(63\gamma^2+206\gamma-84)\theta_u^2-(597\gamma+56)\theta_u)}{4A} \quad (46)$$

$$v^* = \frac{(1-\gamma\theta_u)(492-8\gamma^3\theta_u^5+(9\gamma^2-48\gamma^3)\theta_u^4+(120\gamma^3-118\gamma^2+125\gamma)\theta_u^3+(173\gamma^2+298\gamma-180)\theta_u^2-(935\gamma-72)\theta_u)}{4A} \quad (47)$$

$$\bar{w}_1^* = \frac{1}{32A}\left(\begin{array}{l}10192+118\gamma^4\theta_u^6+(588\gamma^4-340\gamma^3)\theta_u^5+(-1570\gamma^4-1784\gamma^3-582\gamma^2)\theta_u^4+(4460\gamma^3-348\gamma^2+2804\gamma)\theta_u^3\\ +(3074\gamma^2+3816\gamma-2352)\theta_u^2+(-16508\gamma-1568)\theta_u\end{array}\right) \quad (48)$$

$$w_1^* = \frac{1}{32A}\left(\begin{array}{l}9096+75\gamma^3\theta_u^6+(235\gamma^3-91\gamma^2)\theta_u^5-(1895\gamma^3-189\gamma^2+742\gamma)\theta_u^4+(2225\gamma^3+375\gamma^2+90\gamma+1080)\theta_u^3\\ -(729\gamma^2-8526\gamma+1896)\theta_u^2+(-11330\gamma-5208)\theta_u\end{array}\right) \quad (49)$$

$$\bar{q}_{1n}^* = \frac{\gamma^2\theta_u^4-54\gamma^2\theta_u^3+85\gamma^2\theta_u^2+37\gamma\theta_u^3+138\gamma\theta_u^2-367\gamma\theta_u-60\theta_u^2-40\theta_u+260}{4A} \quad (50)$$

$$q_{1n}^* = \frac{(1-\gamma\theta_u)(132+5\gamma^2\theta_u^3+(17\gamma-25\gamma^2)\theta_u^2-(29\gamma+36)\theta_u)}{2A} \quad (51)$$

$$\bar{w}_2^* = \frac{364-9\gamma^3\theta_u^5+(50\gamma^2-114\gamma^3)\theta_u^4+(235\gamma^3+196\gamma^2-\gamma)\theta_u^3+(-502\gamma^2+62\gamma-84)\theta_u^2-(141\gamma+56)\theta_u}{4A} \quad (52)$$

$$w_2^* = \frac{492-15\gamma^3\theta_u^5+(2\gamma^2-30\gamma^3)\theta_u^4+(125\gamma^3+20\gamma^2+149\gamma)\theta_u^3+(-54\gamma^2+26\gamma-180)\theta_u^2-(607\gamma-72)\theta_u}{4A} \quad (53)$$

$$\bar{q}_{2n}^* = \frac{214-6\gamma^3\theta_u^5+(-36\gamma^3+13\gamma^2)\theta_u^4+(90\gamma^3+54\gamma^2+37\gamma)\theta_u^3+(-139\gamma^2+22\gamma-66)\theta_u^2+(4-187\gamma)\theta_u}{4A} \quad (54)$$

$$q_{2n}^* = \frac{214-6\gamma^3\theta_u^5+(-36\gamma^3+13\gamma^2)\theta_u^4+(90\gamma^3+54\gamma^2+37\gamma)\theta_u^3+(-139\gamma^2+22\gamma-66)\theta_u^2+(4-187\gamma)\theta_u}{4A} \quad (55)$$

$$\bar{q}_v^* = q_{2p}^* = \frac{64+3\gamma^2\theta_u^4-(42\gamma^2-27\gamma)\theta_u^3+(55\gamma^2+46\gamma-48)\theta_u^2-(169\gamma-64)\theta_u}{8A} \quad (56)$$

$$\Pi_1^* = \frac{2624-13\gamma^3\theta_u^5+(-158\gamma^3+30\gamma^2)\theta_u^4+(395\gamma^3-268\gamma^2+235\gamma)\theta_u^3+(318\gamma^2+1378\gamma-384)\theta_u^2-(3469\gamma+688)\theta_u}{16A} \quad (57)$$

where $A = 312-5\gamma^3\theta_u^5-\gamma^2(30\gamma-7)\theta_u^4+\gamma(75\gamma^2+2\gamma+48)\theta_u^3-(25\gamma^2-120\gamma+72)\theta_u^2-(384\gamma+48)\theta_u$. The expressions of intermediaries' profit functions are tedious and omitted for brevity; but available from the authors upon request.

## Appendix B: Proofs of Propositions and Corollaries

In this section, to facilitate our discussion, we refer to Appendix A when necessary. Throughout the analysis when finding the signs of expressions, we consider the bounded support of the three problem parameters (i.e., $\beta, \gamma, \theta_u \in [0,1]$).

*Proof of Proposition 1.* Under precommitted buyback pricing, consider solution 1 where buyback quantity is positive, i.e., $\bar{q}_v^* = q_{2p}^* > 0$. Using backward induction, we substitude the second period quantities and prices (i.e., (15)-(18) and (20)-(21)) in first period objective functions (P4) and (P5) (see §3.1.2) for the dealer and rental agency, respectively. Then we solve the intermediaries' profit maximization problems simultaneously and find the first period order quantities in terms of the first period wholesale prices as follows:

$$\bar{q}_{1n}^* = \frac{\begin{array}{l}-4\gamma\theta_u(1-\theta_u)(1-\gamma\theta_u)(2-\gamma\theta_u)w_1+(2-\gamma\theta_u)(16\gamma^2\theta_u^3-32\gamma^2\theta_u^2-11\gamma\theta_u^3-42\gamma\theta_u^2+117\gamma\theta_u+20\theta_u^2+24\theta_u-108)\bar{w}_1\\ +(216-3\gamma^2\theta_u^4-42\gamma^2\theta_u^3+93\gamma^2\theta_u^2+32\gamma\theta_u^3+96\gamma\theta_u^2-288\gamma\theta_u-40\theta_u^2-48\theta_u)\bar{v}\\ +216-16\gamma^4\theta_u^5+32\gamma^4\theta_u^4+12\gamma^3\theta_u^5+60\gamma^3\theta_u^4-160\gamma^3\theta_u^3-43\gamma^2\theta_u^4-102\gamma^2\theta_u^3+345\gamma^2\theta_u^2+62\gamma\theta_u^3+108\gamma\theta_u^2-410\gamma\theta_u-40\theta_u^2-48\theta_u\end{array}}{2A}$$

$$q_{1n}^* = \frac{\begin{array}{l} 8(1-\gamma\theta_u)(2-\gamma\theta_u)(\gamma^3\theta_u^3 - 3\gamma^2\theta_u^2 + 5\gamma\theta_u - 4)w_1 + 2\gamma\theta_u(1-\theta_u)(5-3\gamma\theta_u)(1-\gamma\theta_u)(2-\gamma\theta_u)\bar{w}_1 \\ +(6\gamma^4\theta_u^4 - 6\gamma^4\theta_u^5 + 34\gamma^3\theta_u^4 - 34\gamma^3\theta_u^3 - 64\gamma^2\theta_u^3 + 64\gamma^2\theta_u^2 + 40\gamma\theta_u^2 - 40\gamma\theta_u)\bar{v} \\ -(1-\gamma\theta_u)(8\gamma^4\theta_u^5 - 16\gamma^4\theta_u^4 - 36\gamma^3\theta_u^4 + 76\gamma^3\theta_u^3 + 71\gamma^2\theta_u^3 - 159\gamma^2\theta_u^2 - 80\gamma\theta_u^2 + 192\gamma\theta_u + 40\theta_u - 104) \end{array}}{(1-\gamma\theta_u)A}$$

where $A = 216 - 16\gamma^4\theta_u^5 + 32\gamma^4\theta_u^4 + 8\gamma^3\theta_u^5 + 64\gamma^3\theta_u^4 - 152\gamma^3\theta_u^3 - 29\gamma^2\theta_u^4 - 118\gamma^2\theta_u^3 + 323\gamma^2\theta_u^2 + 52\gamma\theta_u^3 + 120\gamma\theta_u^2 - 396\gamma\theta_u - 40\theta_u^2 - 48\theta_u \geq 0$. Taking derivatives of expression above for $\bar{q}_{1n}$ and $q_{1n}$ we find
*i.* $\frac{\partial q_{1n}^*}{\partial \bar{v}} < 0$, $\frac{\partial q_{1n}^*}{\partial \bar{w}_1} > 0$, $\frac{\partial q_{1n}^*}{\partial w_1} < 0$, and *ii.* $\frac{\partial \bar{q}_{1n}^*}{\partial \bar{v}} > 0$, $\frac{\partial \bar{q}_{1n}^*}{\partial \bar{w}_1} < 0$, $\frac{\partial \bar{q}_{1n}^*}{\partial w_1} < 0$ . □

*Proof of Proposition 2.* In order to show that under precommitted buyback pricing, when the equilibrium buyback quantity is positive (i.e., $\bar{q}_v^* = q_{2p}^* > 0$), the manufacturer's first period cumulative profit $\Pi_1^*$ is submodular in $(\bar{v}, w_1)$, we first substitude $\bar{q}_{1n}^*$ and $q_{1n}^*$ from the proof of Proposition 1 in $\Pi_1$ given in (P6) (see §3.1.2). Then we calculate $\frac{\partial^2 \Pi_1}{\partial \bar{v} \partial w_1}$ as

$$\frac{\partial^2 \Pi_1}{\partial \bar{v} \partial w_1} = \frac{2(1-\theta_u)(2-\gamma\theta_u)\left(\begin{array}{l} 32(\theta_u^7 - 2\theta_u^6)\gamma^6 - (22\theta_u^7 + 228\theta_u^6 - 522\theta_u^5)\gamma^5 + (160\theta_u^6 + 576\theta_u^5 - 1632\theta_u^4)\gamma^4 - (455\theta_u^5 + 562\theta_u^4 - 2489\theta_u^3)\gamma^3 \\ +(673\theta_u^4 - 50\theta_u^3 - 1919\theta_u^2)\gamma^2 + (-548\theta_u^3 + 488\theta_u^2 + 668\theta_u)\gamma + 200\theta_u^2 - 272\theta_u - 56\gamma\theta_u \end{array}\right)}{(1-\gamma\theta_u)(16(\theta_u^5 - 2\theta_u^4)\gamma^4 - (8\theta_u^5 + 64\theta_u^4 - 152\theta_u^3)\gamma^3 + (29\theta_u^4 + 118\theta_u^3 - 323\theta_u^2)\gamma^2 - (52\theta_u^3 + 120\theta_u^2 - 396\theta_u)\gamma + 40\theta_u^2 + 48\theta_u - 216)^2} \tag{58}$$

which is a function of only two parameters $\theta_u$ and $\gamma$. Therefore, plotting $\frac{\partial^2 \Pi_1}{\partial \bar{v} \partial w_1}$ with respect to $\theta_u$ and $\gamma$ in Figure 6 helps us show that the expression is nonpositive in the region of interest. Futhermore, we also maximize (58) s.t. $0 \leq \gamma \leq 1$, $0 \leq \theta_u \leq 1$ using NLPSolve function of Maple software and find the maximum as 0. Since the maximum value the expression in (58) can take is zero and from Figure 6, we conclude that $\frac{\partial^2 \Pi_1}{\partial \bar{v} \partial w_1} \leq 0$; and therefore $\Pi_1^*$ is submodular in $(\bar{v}, w_1)$.

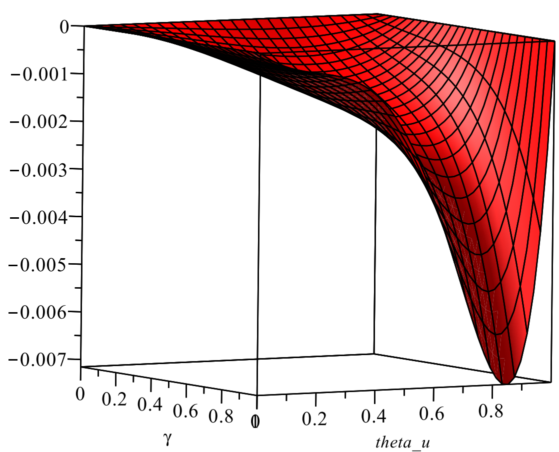


**Figure 6** **Plot of** $\frac{\partial^2 \Pi_1}{\partial \bar{v} \partial w_1}$ **given in** (58).

Next, we calculate $\frac{\partial^2 \Pi_1}{\partial \bar{v} \partial \bar{w}_1}$ as:

$$\frac{\partial^2 \Pi_1}{\partial \bar{v} \partial \bar{w}_1} = \frac{\left(\begin{array}{l} (128\theta_u^{10} - 512\theta_u^9 + 512\theta_u^8)\gamma^8 - (80\theta_u^{10} + 1088\theta_u^9 - 5232\theta_u^8 + 5472\theta_u^7)\gamma^7 + (17\theta_u^{10} + 676\theta_u^9 + 3782\theta_u^8 - 22972\theta_u^7 + 25153\theta_u^6)\gamma^6 \\ -(128\theta_u^9 + 2224\theta_u^8 + 5568\theta_u^7 - 49808\theta_u^6 + 57632\theta_u^5)\gamma^5 + (290\theta_u^8 + 1904\theta_u^7 + 2924\theta_u^6 - 39600\theta_u^5 + 47922\theta_u^4)\gamma^4 \\ +(397\theta_u^7 + 5772\theta_u^6 - 8114\theta_u^5 - 40564\theta_u^4 + 61453\theta_u^3)\gamma^3 + (-2564\theta_u^6 - 15312\theta_u^5 + 30056\theta_u^4 + 109744\theta_u^3 - 178756\theta_u^2)\gamma^2 \\ +(3560\theta_u^5 + 13152\theta_u^4 - 37008\theta_u^3 - 82080\theta_u^2 + 153576\theta_u)\gamma - 1600\theta_u^4 - 3840\theta_u^3 + 14976\theta_u^2 + 20736\theta_u - 46656 \end{array}\right)}{(\gamma\theta_u - 1)(16(\theta_u^5 - 2\theta_u^4)\gamma^4 - (8\theta_u^5 + 64\theta_u^4 - 152\theta_u^3)\gamma^3 + (29\theta_u^4 + 118\theta_u^3 - 323\theta_u^2)\gamma^2 - (52\theta_u^3 + 120\theta_u^2 - 396\theta_u)\gamma + 40\theta_u^2 + 48\theta_u - 216)^2} \tag{59}$$

which is a function of only two parameters $\theta_u$ and $\gamma$. Therefore, plotting $\frac{\partial^2 \Pi_1}{\partial \bar{v} \partial \bar{w}_1}$ with respect to $\theta_u$ and $\gamma$ in Figure 7 helps us show that the expression is always positive in the region of interest. Futhermore, we also minimize (58) s.t. $0 \le \gamma \le 1$, $0 \le \theta_u \le 1$ using NLPSolve function of Maple software and find the minimum as 1. Since the minimum value the expression in (58) can take is zero and from Figure 7, we conclude that $\frac{\partial^2 \Pi_1}{\partial \bar{v} \partial \bar{w}_1} \ge 0$; and therefore $\Pi_1^*$ is supermodular in $(\bar{v}, \bar{w}_1)$. $\square$

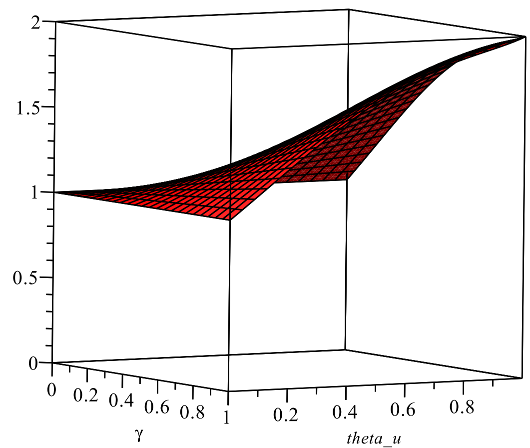


**Figure 7** **Plot of $\frac{\partial^2 \Pi_1}{\partial \bar{v} \partial \bar{w}_1}$ given in** (59).

*Proof of Proposition 3.* Consider precommitted buyback pricing regime. In Appendix A, presenting the characterization of the monopoly solution, we show that zero-buyback outcome arises $\forall$ such that $\bar{v} \le \bar{v}_T$ where $\bar{v}_T$ is defined in (42). Under positive-buyback outcome, optimal buyback price $\bar{v}^*$ is given in (43) (see Appendix A.2). Furthermore, $\bar{v}^* - \bar{v}_T \ge 0$. Finally, $\frac{\partial \bar{v}_T}{\partial \gamma} = -\frac{\theta_u(1-\gamma\theta_u)(33-3\gamma^3\theta_u^3-3\gamma^2\theta_u^2+13\gamma\theta_u)}{8(3-\gamma^2\theta_u^2)^2} < 0$ and $\frac{\partial \bar{v}_T}{\partial \theta_u} = -\frac{\gamma(1-\gamma\theta_u)(33-3\gamma^3\theta_u^3-3\gamma^2\theta_u^2+13\gamma\theta_u)}{8(3-\gamma^2\theta_u^2)^2} < 0$. $\square$

*Proof of Proposition 4.* Under precommitted buyback pricing, the optimal profits under zero- and positive-buyback outcomes are given in (41) and (45), respectively (see Appendix A.1 and A.2). Let $\Delta$ denote the difference between the two expressions, i.e., $\Delta =$ (45) - (41). Maximizing $\Delta$ s.t. $0 \le \gamma \le 1$, $0 \le \theta_u \le 1$ gives $\Delta^* = 0$ at $\gamma = 1$, $\theta_u = 1$. Therefore $\Delta \le 0$ and thus (45) $\le$ (41) ; i.e., the manufacturer is better off under zero-buyback outcome which arises when $\bar{v}^* \le \bar{v}_T$. $\square$

*Proof of Corollary 1.* Consider positive buyback outcome solution (i.e., $\bar{q}_v^* > 0$) under precommitted buyback pricing given in Appendix A.2. Buyback price $\bar{v}^*$ is given (43) and the resale price $v^*$ is given in (44). The difference $\bar{v}^* - v^* \ge 0$. $\square$

*Proof of Proposition 5.* Under postponed buyback pricing, consider solution 1 where buyback quantity is positive, i.e., $\bar{q}_v^* = q_{2p}^* > 0$. Using backward induction, we substitude the second period quantities and prices (given in (15)-(18) and (19)-(20)) as well as the buyback price $\bar{v}^*$ (given in (22)) in first period objective functions (P4) and (P5) for the dealer and rental agency, respectively. Then we solve the profit maximization problem of the intermediaries' simultaneously and find the first period order quantities in terms of the first period wholesale prices as follows:

$$q_{1n}^* = \frac{4(1-\gamma\theta_u)((6\gamma^2\theta_u^2+18\gamma\theta_u-56)w_1+5\theta_u\gamma(1-\theta_u)\bar{w}_1-6\gamma^2\theta_u^2+20\gamma\theta_u^2-38\theta_u\gamma-35\theta_u+91)}{A} \tag{60}$$

$$\bar{q}_{1n}^* = \frac{2}{A}\begin{pmatrix} 2\gamma\theta_u(1-\theta_u)(1-\gamma\theta_u)w_1-(11\gamma\theta_u^3+42\gamma\theta_u^2-117\gamma\theta_u-20\theta_u^2-24\theta_u+108)\bar{w}_1-7\gamma^2\theta_u^4-36\gamma^2\theta_u^3+91\gamma^2\theta_u^2+33\gamma\theta_u^3 \\ +96\gamma\theta_u^2-289\theta_u\gamma-35\theta_u^2-42\theta_u+189 \end{pmatrix} \tag{61}$$

where $A=756-7\gamma^3\theta_u^5-34\gamma^3\theta_u^4+89\gamma^3\theta_u^3-11\gamma^2\theta_u^4-74\gamma^2\theta_u^3+181\gamma^2\theta_u^2+122\gamma\theta_u^3+348\gamma\theta_u^2-1062\gamma\theta_u-140\theta_u^2-168\theta_u\geq 0$. Taking the derivatives of expression for $\bar{q}_{1n}^*$ and $q_{1n}^*$ above we find *i.* $\frac{\partial q_{1n}^*}{\partial \bar{w}_1}>0$ , $\frac{\partial q_{1n}^*}{\partial w_1}<0$ and *ii.* $\frac{\partial \bar{q}_{1n}^*}{\partial \bar{w}_1}>0$, $\frac{\partial \bar{q}_{1n}^*}{\partial w_1}<0$ . $\square$

*Proof of Proposition 6.* Consider postponed buyback pricing regime. In Appendix A, presenting the characterization of the monopoly solution, we show that zero-buyback outcome arises $\forall$ such that $\bar{v}\leq\bar{v}_T$ where $\bar{v}_T$ is defined in (42). Under positive-buyback outcome, optimal buyback price $\bar{v}^*$ is given in (46). Furthermore, $\bar{v}^*-\bar{v}_T\geq 0$. $\square$

*Proof of Proposition 7.* For postponed buyback pricing, the optimal profits under zero- and positive- buyback outcomes are given in (41) and (57), respectively. Let $\Delta$ denote the difference between the two expressions, i.e., $\Delta=$ (57) - (41). Minimizing $\Delta$ s.t. $0\leq\gamma\leq 1$, $0\leq\theta_u\leq 1$ gives $\Delta^*=1.85310^{-7}$. Therefore $\Delta\geq 0$ and thus (57) $\geq$ (41). Under positive-buyback outcome, optimal buyback price $\bar{v}^*$ is given in (46) and resale price $v^*$ is given in (47). Here, (46) < (47). $\square$

*Proof of Proposition 8.* From Proposition 7, we know that postponed buyback pricing yields positive buyback quantity (i.e., $q_{2p}^*>0$) and from Proposition 4 we know that precommitted buyback pricing yields zero buyback quantity (i.e., $q_{2p}^*=0$). To compare the profits under two strategies, let $\Delta=\Pi_1^{N*}-\Pi_1^{C*}$. Minimizing $\Delta$ s.t. $0\leq\gamma\leq 1$, $0\leq\theta_u\leq 1$ gives $\Delta^*=1.85310^{-7}$. Therefore, $\Delta\geq 0$ and $\Pi_1^{N*}\geq\Pi_1^{C*}$. $\square$

*Proof of Proposition 9.* Consider (C,C) equilibrium when the buyback quantities are positive, i.e., $\bar{q}_v^{a*}=\bar{q}_v^{b*}>0$. Then substituting second period quantities given in (28)-(30) in manufacturers' objective functions given in (P3) (see §3.1.1 and recall that the manufacturer problem remains the same under competition as in under monopoly) and solving FOCs (i.e., $\frac{\partial\Pi_1^a}{w_2^a}=0$, $\frac{\partial\Pi_1^a}{\bar{w}_2^a}=0$, $\frac{\partial\Pi_1^b}{w_2^b}=0$, $\frac{\partial\Pi_1^b}{\bar{w}_2^b}=0$) we find the second period wholesale prices as follows:

$$\bar{w}_2^{a*}=\frac{\begin{pmatrix} 2\gamma\theta_u(1-\theta_u)(2-\beta^2-\gamma\theta_u)q_{1n}^a+\gamma\theta_u\beta(1-\theta_u)(3-\beta^2-2\gamma\theta_u)q_{1n}^b+8\gamma\theta_u(1-\gamma\theta_u)(2-\beta^2-\gamma\theta_u)\bar{q}_{1n}^a+4\gamma\theta_u\beta(1-\gamma\theta_u)(3-\beta^2-2\gamma\theta_u)\bar{q}_{1n}^b \\ +(\beta^4+2\beta^2\gamma\theta_u-9\beta^2-8\gamma\theta_u+16)\bar{v}_a-2\gamma\theta_u\beta\bar{v}_b+\gamma\theta_u(1-\beta^2)(\beta^2+2\gamma\theta_u+\beta-4) \end{pmatrix}}{(4-2\gamma\theta_u-\beta^2+\beta)(4-2\gamma\theta_u-\beta^2-\beta)} \tag{62}$$

$$\bar{w}_2^{b*}=\frac{\begin{pmatrix} 2\gamma\theta_u(1-\theta_u)(2-\beta^2-\gamma\theta_u)q_{1n}^b+\gamma\theta_u\beta(1-\theta_u)(3-\beta^2-2\gamma\theta_u)q_{1n}^a+8\gamma\theta_u(1-\gamma\theta_u)(2-\beta^2-\gamma\theta_u)\bar{q}_{1n}^b+4\gamma\theta_u\beta(1-\gamma\theta_u)(3-\beta^2-2\gamma\theta_u)\bar{q}_{1n}^a \\ +(\beta^4+2\beta^2\gamma\theta_u-9\beta^2-8\gamma\theta_u+16)\bar{v}_b-2\gamma\theta_u\beta\bar{v}_a+\gamma\theta_u(1-\beta^2)(\beta^2+2\gamma\theta_u+\beta-4) \end{pmatrix}}{(4-2\gamma\theta_u-\beta^2+\beta)(4-2\gamma\theta_u-\beta^2-\beta)} \tag{63}$$

while the new-car wholesale prices (i.e., $w_2^{k*}=\frac{1-(1-\theta_u)q_{1n}^k}{2}$ for $k\in\{a,b\}$) remain the same as in the monopoly setting, i.e., as given in equation (20). Therefore, we conclude that $\frac{\partial\bar{w}_2^{a*}}{\partial q_{1n}^a}>0$, $\frac{\partial\bar{w}_2^{a*}}{\partial\bar{q}_{1n}^a}>0$, $\frac{\partial\bar{w}_2^{a*}}{\partial\bar{v}^a}>0$, $\frac{\partial\bar{w}_2^{a*}}{\partial q_{1n}^b}>0$, $\frac{\partial\bar{w}_2^{a*}}{\partial\bar{q}_{1n}^b}>0$, and $\frac{\partial\bar{w}_2^{a*}}{\partial\bar{v}^b}<0$. $\square$

*Proof of Proposition 10.* Consider (C,C) equilibrium when the buyback quantities are positive, i.e., $\bar{q}_v^{a*}=\bar{q}_v^{b*}>0$. First substituting second period rental-car wholesale prices given in (62)-(63) and new-car wholesale prices given in (20) in intermediaries' second period problems in (P4) (see §3.1.2 and recall that the dealer problem remains the same under competition as in under monopoly) and (P8) (see §5.1.1); and then solving FOCs we find the first-period order quantities. The expressions of quantities as well as the derivatives are tedious and available from the authors upon request; however omitted here for brevity. In sum, we find that: 1. Rental agency's order quantity depends on the buyback prices and first-period prices as follows: $\frac{\partial \bar{q}_{1n}^{a*}}{\partial \bar{v}^a} > 0$, $\frac{\partial \bar{q}_{1n}^{a*}}{\partial \bar{v}^b} < 0$, $\frac{\partial \bar{q}_{1n}^{a*}}{\partial \bar{w}_1^a} < 0$, $\frac{\partial \bar{q}_{1n}^{a*}}{\partial w_1^a} < 0$, $\frac{\partial \bar{q}_{1n}^{a*}}{\partial \bar{w}_1^b} > 0$, and $\frac{\partial \bar{q}_{1n}^{a*}}{\partial w_1^b} > 0$; 2. Dealer a's order quantity depends on the buyback prices and first-period prices as follows: $\frac{\partial q_{1n}^{a*}}{\partial \bar{v}^a} < 0$, $\frac{\partial q_{1n}^{a*}}{\partial \bar{v}^b} > 0$, $\frac{\partial q_{1n}^{a*}}{\partial \bar{w}_1^a} > 0$, $\frac{\partial q_{1n}^{a*}}{\partial w_1^a} < 0$, $\frac{\partial q_{1n}^{a*}}{\partial \bar{w}_1^b} < 0$, and $\frac{\partial q_{1n}^{a*}}{\partial w_1^b} > 0$.

Finally, using the same approach we find the changes in first period quantities with respect to prices under (N,N) equilibrium as shown in Table 4 and under asymmetric equilibria (i.e., (N,C) and (C,N)) as shown in Table 6.

| | | **(C,N) Equilibrium** | | | | | | **(N,C) Equilibrium** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\bar{w}_1^a$ | $\bar{w}_1^b$ | $w_1^a$ | $w_1^b$ | $\bar{v}^a$ | $\bar{v}^b$ | $\bar{w}_1^a$ | $\bar{w}_1^b$ | $w_1^a$ | $w_1^b$ | $\bar{v}^a$ | $\bar{v}^b$ |
| $\bar{q}_v^{a*} > 0,\ \bar{q}_v^{b*} > 0$ | $q_{1n}^a$ | + | - | - | + | - | NA | + | - | - | + | NA | + |
| | $q_{1n}^b$ | - | + | + | - | + | NA | - | + | + | - | NA | - |
| | $\bar{q}_{1n}^a$ | - | + | - | - | + | NA | - | + | + | + | NA | - |
| | $\bar{q}_{1n}^b$ | + | - | + | + | - | NA | + | - | - | - | NA | + |
| $\bar{q}_v^{a*} > 0,\ \bar{q}_v^{b*} = 0$ | $q_{1n}^a$ | + | - | - | 0 | - | NA | + | + | - | 0 | NA | 0 |
| | $q_{1n}^b$ | 0 | 0 | 0 | - | 0 | NA | 0 | 0 | 0 | - | NA | 0 |
| | $\bar{q}_{1n}^a$ | - | + | - | 0 | + | NA | - | + | + | 0 | NA | 0 |
| | $\bar{q}_{1n}^b$ | + | - | + | 0 | - | NA | + | - | + | 0 | NA | 0 |
| $\bar{q}_v^{a*} = 0,\ \bar{q}_v^{b*} = 0$ | $q_{1n}^a$ | 0 | 0 | - | 0 | 0 | NA | 0 | 0 | - | 0 | NA | 0 |
| | $q_{1n}^b$ | 0 | 0 | 0 | - | 0 | NA | 0 | 0 | 0 | - | NA | 0 |
| | $\bar{q}_{1n}^a$ | - | + | 0 | 0 | 0 | NA | - | + | 0 | 0 | NA | 0 |
| | $\bar{q}_{1n}^b$ | + | - | 0 | 0 | 0 | NA | + | - | 0 | 0 | NA | 0 |
| $\bar{q}_v^{a*} = 0,\ \bar{q}_v^{b*} > 0$ | $q_{1n}^a$ | 0 | 0 | - | 0 | 0 | NA | 0 | 0 | - | 0 | NA | 0 |
| | $q_{1n}^b$ | + | + | 0 | - | 0 | NA | - | + | 0 | - | NA | - |
| | $\bar{q}_{1n}^a$ | - | + | 0 | + | 0 | NA | - | + | 0 | + | NA | - |
| | $\bar{q}_{1n}^b$ | + | - | 0 | + | 0 | NA | + | - | 0 | - | NA | + |

**Table 6** **Direction of change in first-period quantities with respect to prices under C-N and N-C equilbria**

□